Introduction to Game Theory II

Components of Games, Strategies, players, information sets, types of strategies, different ways to represent games.

A Conceptual Outline

- I divide the analysis of games into two major parts.
- First, we analyze various ways of representing a game:
  - Game trees, bimatrix games, mathematical games.
  - Starts in this lecture and goes on to “An Introduction to Games and Their Theory III: Payoffs, profits, expected utility, common knowledge (DS Ch. 2 and Ch. 5, Appendix)”
A Conceptual Outline

Second, we “solve” the game (that is, predict how it will be played:
- Backward induction, Dominant solvable, Nash Equilibrium.
- Starting in “An Introduction to Games and Their Theory IV: Best Responses (DS Ch. 2 and Ch. 5, Appendix)” and beyond.

Lecture 2: General Concepts

- Outline of lecture
  - Components of a game
  - Multiple ways to represent a game: game trees; matrix games; mathematical forms.
  - Strategies: definitions, different types, contingent vs. non-contingent
  - examples.
2: General Concepts

• Components of a game:
  – A game consists of three main things:
    • a list of players;
    • a (complete) description of the strategies of each player;
    • a description of the consequences (payoffs) for each player for every possible profile of strategy choices of all players.

• Despite this simple triplet, games appear in a wide variety of forms:
  – game trees
  – matrix games
  – mathematical descriptions.
  – Each representation must be capable of showing the three main components of a game.
2: General Concepts

• Game trees:
  – game trees are very useful when players choose strategies sequentially.
  – examples include:
    • the centipede game
    • offer/counter offer bargaining games
    • market games with a market “leader”

The Centipede Game

```
1   2   1   2   1   2   ...  
1.0 0.2 0.4 0.6 0.8 1.0 0.2
```

2: General Concepts

- Matrix games are used for two player, simultaneous move games.
  - examples include:
    - prisoner’s dilemma games
    - “battle of the sexes” game
    - coordination games.

Battle of the Networks, normal form: The payoff matrix

<table>
<thead>
<tr>
<th></th>
<th>Network 1</th>
<th>Network 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sitcom</td>
<td>55%, 45%</td>
<td>52%, 48%</td>
</tr>
<tr>
<td>Sports</td>
<td>50%, 50%</td>
<td>45%, 55%</td>
</tr>
</tbody>
</table>
2: General Concepts

• Mathematical representations are used for more complicated games:
  – examples include:
    • Cournot games of quantity setting oligopolists
    • Bertrand games of price setting oligopolists
    • auction games.

Example of a Mathematical Representation--Cournot Game

• Players are Firm 1 and Firm 2
• Strategies are $Q_1$ and $Q_2$ which are positive real numbers (quantity each firm places on market to be sold.)
• payoffs are profits:
  – Profits($Q_1, Q_2$) = $P(Q_1, Q_2) * Q_1 - C(Q_1)$. 
Definitions

- **Strategy**: a complete plan of action (what to do in *every* contingency) (that is, at every node)
- **Information Set**: for player *i* is a collection of decision nodes satisfying two conditions: player *i* has the move at every node in the collection, and *i* doesn't know which node in the collection has been reached
- (Used in extensive form games.)

Game 3: How many info sets?

```
  1
 /  \
L    R
 2  2
 l  r l  r
 1  0 3 -1 2
 2  2 1 -1 0
```
Game 4: How many info sets?

More Definitions

- **Perfect Information**: each information set is a single node (Chess, checkers, go, ...)
- Finite games of perfect information can be "solved" by "rollback" (backward induction) in the extensive form or elimination of weakly dominated strategies in the normal form
- **Imperfect Information**: at some point in the tree some player is not sure of the complete history of the game so far
Game 3: Backward Induction
Looking ahead and reasoning back

Different Types of Strategies

- strategies may be a simple collection of **discrete** choices:
  - \{Up, Down\}, \{In, Out\} as in matrix games.
Different Types of Strategies

• they can be **contingent** strategies that say what will happen if a given information set is reached:
  – {Right if Down occurs, Middle if Up occurs} as in game trees.

• They can also be **continuous** variables as in pricing games: $p = .5$. 