Instructions. Do All Questions. Each question has equal weight. PLEASE WRITE CLEARLY AND CONCISELY! LIMIT: 5 PAGES per question. The exam may be picked up on Tuesday June 10 and MUST be returned to Anna by SUNDAY JUNE 15.

1. Consider an overlapping generations model in which each generation lives for three periods, young, middle-aged, and old, and the population grows at a constant rate \( n \). There is no altruism across generations and individuals work in the first two periods of life and are retired in the last period of life. They can invest their saving in the world capital market at a given rate of return \( r \). Within each generation, labor income is heterogeneous, in that individuals differ in the amount of (effective) time they have to allocate to labor \( l^i \) or leisure \( x^i \) according to 
\[
1 + e^i = l^i + x^i,
\]
where the individual productivity parameters \( e^i \) are distributed according to a known distribution function. The government finances a pension scheme with lump sum transfers \( f \) to the old generation with a proportional tax, \( \tau \), on labor. The government budget constraint is:
\[
f = \tau l^Y (1 + n)^2 + \tau l^M (1 + n).
\]
The subjective discount rate \( \delta \) equals the real interest rate \( \rho \), and all individuals may save assets at the real interest rate. A young individual \( i \)'s lifetime utility from the pension scheme is:
\[
\omega^Y = U(c^Y) + \frac{1}{1 + \delta} U'(c^M) + \frac{1}{(1 + \delta)^2} c^{IO} + V(x^Y) + \frac{1}{1 + \delta} V(x^M),
\]
and a young individual’s intertemporal budget constraint is:
\[
c^Y + c^M \frac{1}{1 + \rho} + c^{IO} \frac{1}{(1 + \rho)^2} = l^Y (1 - \tau) + \frac{l^M (1 - \tau)}{1 + \rho} + \frac{f}{(1 + \rho)^2}.
\]

a. Solve for the individual’s optimal consumption path and labor supply.

b. What is the total present value of the pension scheme to a young person of productivity \( e^i \)? What is the pension scheme’s net value to a young person of average productivity? How much larger is the net present value of an individual of productivity \( e^m < e \) relative to that of an individual of average productivity? Describe how these two values relate to redistribution between and within generations. Write down the equation describing the tax rate preferred by a young individual with productivity \( e^i \) and relate the terms in this expression to the above discussion.
c. Suppose that productivity is higher for middle-aged individuals than for young individuals. In particular, a young individual with productivity \( e^{iY} \) will achieve productivity:

\[
e^{iM} = e^{iY} + \frac{(1+n)(2+n)}{2+\rho} [L(\tau^i) + \tau L(\tau^i)]
\]

when middle-aged (\( \tau^i \) is the tax rate preferred by the individual with productivity \( e^{iY} \)) and \( L(\tau) \equiv 1 + e - V_x^{-1}(1-\tau) \) is decreasing in \( \tau \). Therefore, the share of young voters with productivity lower than \( e^{iY} \) equals the share of young voters with productivity lower than:

\[
e^{iY} + \frac{(1+n)(2+n)}{2+\rho} [L(\tau) + \tau L(\tau)]
\]

The distribution of productivities is \( F(e^{iY}) \) for the young.

Show how the share of young and middle-aged voters who support higher taxes in equilibrium depends on \( n \). Discuss how large a share of the young and middle-aged voters will support higher taxes in equilibrium when \( n = 0 \) and when \( n \) becomes very large.

d. (Harder) Suppose that voting rights exclude people with very low incomes. The people excluded from the franchise exist in equal proportion among the young, middle-aged, and old. Describe how the equilibrium tax rate would change. Discuss the relation to within- and between-generations redistribution.

2. Describe carefully the basic components and the key results (either mathematically or with a careful verbal description) of one opportunistic or partisan model of the political business cycle. For the model you have chosen, explain what are its strengths and weaknesses relative to other models of the political business cycle, both theoretically and empirically?

3. Discuss in detail one important real-world example of a non-optimal major policy, a failure to adopt a policy change, or a delay in adopting a major policy change (but do not use an example if it was discussed in detail in class) and how the failure can be explained in terms of one of the models discussed in class. Your essay should include (but not necessarily be limited to):

1) A background discussion of what the economic problem was and what exactly was the failure in terms of policy choice;

2) A short summary of what you think the correct policy choice would have been;

3) An intuitive discussion of why there was a failure to adopt the correct policy;

4) Use of one of the models of reform that we discussed to make your discussion in 3) more formal, with a discussion of the assumptions used in building the model, as well as a convincing discussion of why you chose the model you did, as opposed to some other one.