Consider a basic optimal growth problem, a social planner choosing the optimal path of capital accumulation to maximize welfare of the representative individual, where it is assumed that all individuals are identical. Assume constant population, which is set equal to one. Output, which is an increasing, concave function \( f(\cdot) \) of the current capital stock \( k_t \), is divided between current-period consumption \( c_t \) and capital to be carried over to the following period \( k_{t+1} \):

\[
c_t + k_{t+1} = f(k_t).
\]

The planner's objective is to maximize the present discounted value of utility of the individual by choice of sequences of \( c_t \) and \( k_{t+1} \) from \( t = 0 \) to infinity. That is, the planner chooses \( \{c_t, k_{t+1}\} \) to maximize the individual’s present discounted utility:

\[
\Omega = \sum_{t=0}^{\infty} \beta^t u(c_t),
\]

1. Find the optimal solution \( \{k^1, k^2, k^3, \ldots\} \) when \( u(c) = \ln c \) and \( f(k) = k^\alpha \)

2. Suppose now there are two types of individuals with the same instantaneous utility function \( u(\cdot) \) as given above, but different discount rates \( \beta_B \neq \beta_A \). Discuss each group’s optimal time path. What sort of heterogeneity does this represent?

3. How might standard multi-agent welfare economics handle this problem in terms of weighting the agents’ utility in a social welfare function? Be explicit as far as objective functions and constraints. In what sense is the set of solutions Pareto efficient? Discuss the difference between the multi-agent welfare problem and the political economy problem.

4. Use the same framework to represent the case of all agents identical \( ex \ ante \), but with the possibility of \( ex \ post \) heterogeneity.

5. Using this formalization, illustrate conceptually the difference between a political economy approach on the one hand and a public finance and public choice on the other. (For example, public finance might ask how a specific choice in 3 might be decentralized.)