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The scaling of collisionless, magnetic reconnection for large systems

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Abstract

Hybrid simulations with electron inertia, along with analytic scaling arguments, are presented which demonstrate that magnetic reconnection remains Alfvénic in a collisionless system even as the macroscopic scale length of the system becomes very large. This fast reconnection is facilitated by the whistler physics present near the x-line. The reconnection rate is found to be a universal constant corresponding to an inflow velocity towards the x-line of around 0.1 c_A.

Introduction

Magnetic reconnection plays an important role in the dynamics of the magnetosphere, the solar corona, and laboratory experiments by allowing magnetic energy to be released in the form of high velocity streams of electrons and ions. In resistive magnetohydrodynamic (MHD) models of this process, resistivity breaks the frozen-in constraint in a boundary layer called the dissipation region allowing reconnection to occur. At low values of resistivity the dissipation region forms a macroscopic Sweet-Parker layer which severely limits the rate of reconnection [*Sweet,* 1958; *Parker,* 1957; *Biskamp,* 1986], the inflow velocity into the magnetic X-line scaling like

$$v_i \sim \frac{\delta}{L} c_A \ll c_A, \qquad (1)$$

with δ and L being, respectively, the small, resistivity-dependent width and macroscopic length of the dissipation region and c_A being the Alvén velocity.

In collisionless plasma the dissipation region develops a multiscale structure based on electron and ion scale lengths [*Biskamp et al.,* 1997; *Shay et al.,* 1998]. Within a distance from the x-line of the order of the ion inertial length $\delta_i = c/\omega_{pi}$ the ion motion decouples from that of the electrons as a result of the Hall effect [*Sonnerup,* 1979; *Terasawa*, 1983]. In a region of scale size δ_i the ions are accelerated away from the X-line, eventually reaching the Alfvén velocity. Within this ion inertial region but outside of an electron inertial region, the electrons remain frozen-in to the magnetic field and the dynamics of the electron magnetofluid are described by a set of nonlinear whistler equations [*Mandt et al.,* 1994; *Biskamp et al.,* 1997]. Finally, even closer to the X-line the electrons decouple from the magnetic field. Unlike resistive MHD, in a collisionless plasma the mechanism which decouples the electrons from the magnetic field has no effect on the reconnection rate in the nonlinear regime [*Mandt et al.,* 1994; *Biskamp et al.,* 1997; *Hesse et al.,* 1999]. This is a consequence of the quadratic dispersion character of the whistler which controls the dynamics in this inner region [*Shay and Drake,* 1998]. The reconnection rate is insensitive to electron kinetic effects and therefore the electron mass and temperature. The ion dynamics must ultimately limit and control the rate of reconnection.

The determination of the reconnection rate and its scaling is the subject of this paper. The only remaining free parameter, however, is the system size L since the ion temperature T_i has been shown to have no significant affect on the reconnection rate [*Shay et al.,* 1998]. Continuity into and out of the region where the ions are decoupled from the magnetic field (ion-inertial region) yields,

$$v_i \sim (\delta_i/\Delta) c_A, \qquad (2)$$

where $\delta_i = c/\omega_{pi}$ is the width and Δ is the length along the outflow direction of the ion-inertial region. The scaling of Δ with system size, L, is the key factor determining if Alfvénic reconnection is possible for very large systems, i.e., for $L \gg \delta_i$.

In order to examine the effect of system size on Δ, and thus on the quasi-steady reconnection rate, we studied magnetic reconnection in systems with very large scale lengths L using a 2 1/2 dimensional hybrid code with electron inertia. The reconnection rate is independent of L, corresponding to an inflow velocity $v_i \sim 0.1\, c_A$. The whistler physics is the key to understanding these results. Close to the x-line, the fast moving electrons induce a large electric field along the outflow direction which accelerates the ions to their Alfvén speed in a distance $\Delta \sim 10\, \delta_i \ll L$. This result has broad implications, allowing Alfvénic reconnection in the magnetosphere and possibly also in the solar corona.

Simulation Model

The details of the hybrid model have been presented previously and therefore will not be repeated here [*Shay et al.,* 1998]. The ions are treated as particles and the electrons as a finite mass fluid with an isotropic pressure. Time is normalized to the ion cyclotron time Ω_i^{-1} with B evaluated at its maximum initial value. Length scales are normalized to the ion inertial length, $\delta_i \equiv c/\omega_{pi}$, where n is given by the average of the initial density (nearly constant for the simulations presented here). The velocities therefore are normalized to the Alfvén velocity and the temperature to $B^2/(8\pi n)$.

The focus of this work is on the structure of the dissipation region during reconnection and not on the initiation of reconnection during substorm onset. For this reason, we consider an idealized system which exhibits nearly quasi-steady reconnection. The initial configuration of the system is a double current sheet in the (x, z) plane in a box of length L_x and width L_z, with L_x/L_z =(2, 4, or 8) [*Shay and Drake,* 1998]. Small magnetic perturbations and associated currents are added to form seed magnetic islands centered around the two current layers. The grid scales, Δ_x and Δ_z, are $0.5\,\delta_e$. Unless otherwise noted, $\delta_e/\delta_i = 0.2$, corresponding to a mass ratio of 1/25. The magnetic pressure away from the current sheet is balanced by an increase in electron pressure at the current sheet. Unlike a conventional Harris equilibrium in which the variation of the density produces the equilibrium current, we take the density to be nearly constant, the current being produced by the diamagnetic drift of electrons due to a spatially varying T_e. The constant density yields a nearly constant upstream Alfvén velocity, which makes the interpretation of the reconnection rate straightforward. The ion temperature, T_i, is initially spatially uniform and is set to 0.05. There are on average 20 particles/cell initially loaded. The largest simulations are 2048×512 grid points with 20 million particles. The effect of the system size on the reconnection rate is studied by varying L_x and L_z.

Simulation Results

The time dependence of the reconnection rate in this system has been discussed previously [*Shay and Drake,* 1998]. After an initiation phase, the reconnection rate becomes quasi-steady. It is this steady reconnection phase which is of primary interest.

Figure 1 shows various moments for a run with $L_x = 204.8$ and $L_z = 51.2$. at $t = 200\,\Omega_i^{-1}$, which is during the quasi-steady reconnection period. Only a limited area around one of the two x-lines is shown. The separation of scales between the ions and electrons can be clearly seen in Figs. 1c and 1d. The ions form a broad current layer about $2\delta_i$ wide and macroscopic in length with distinct wings which fan outward from the x-line downstream of the separatrix of the magnetic field. The electrons, on the other hand, form a microscopic current sheet about $1\,\delta_e = 0.2\,\delta_i$ wide, and at most a few δ_i long. This sheet splits into wings which very closely map the separatrix. In Fig. 1e, the quadrapole nature of B_y associated with in plane Hall currents and the dynamics of whistlers is readily visible [*Sonnerup,* 1979; *Terasawa*, 1983; *Mandt et al.,* 1994].

Figure 2 shows the reconnection rates from simulations with various values of L_x. The error bars represent the temporal variation of the reconnection rate during the quasi-steady reconnection period. The symbols denote runs with different parameters and aspect ratios: diamonds denote $\delta_e = 0.2$ and $L_x/L_z = 2$; triangles denote $\delta_e = 0.2$ and $L_x/L_z = 4$; squares denote $\delta_e = 0.2$ and $L_x/L_z = 8$; and asterisks denote $\delta_e = 0.1$ and $L_x/L_z = 4$. The reconnection rate is finite and independent of system size for this set of simulations.

It is possible that the reconnection rates shown in Fig. 2 are influenced by the specific geometry of the double current layer. To show that the results are in fact generic, in Fig. 3 we plot the electron out-of-plane current at two different times ($t = 160\,\Omega_i^{-1}$ and $220\,\Omega_i^{-1}$) from the same simulation as in Fig. 1. These correspond to early and late times of the quasi-steady reconnection period. The angle, θ, defining the width of the current wedge directly reflects the rate of reconnection since the $\mathbf{J} \times \mathbf{B}$ force from this current initiates the ion acceleration away from the x-line. This angle is established early in time close to the x-line in Fig. 3a. The wedge then propagates downstream, with θ remaining essentially invariant. The o-line of the adjacent current layer in Fig. 3a is directly above the x-line and has a shape and size comparable to the o-line centered around $x = -150$. That is, it has not yet begun to grow strongly and therefore is not driving the flow into the x-line in Fig. 3a. Thus, the physical processes which control the rate of reconnection are localized around the x-line and the reconnection flow field then propagates both downstream and upstream. A similar result was reported previously [*Lin and Swift,* 1996].

In order to explore the physics of the x-line region, we first examine how the outflow speed of the electrons and ions changes as they flow downstream from the x-line. Fig. 4 is a cut through the x-line along x of the electron and ion velocities for a simulation with $L_x = 102.4$, $L_z = 25.6$, and $\delta_e = 0.1$. Within a distance $10\,\delta_i$ from the x-line is the Hall or whistler region where the electron and ion motion decouples. The electrons accelerate to a velocity greater than the Alfvén speed close to the x-line and then gradually decelerate to join with the ions. The ions continuously accelerate away from the x-line until they reach their maximum outflow speed of about 0.6, which is the Alfvén speed at the inflow edge of the dissipation region. The ions reach their maximum speed within $10\,\delta_i$ of the x-line at the location where they rejoin with the electron flow. Clearly, the physics of the whistler region near the x-line is playing a key role in accelerating the ions away from the x-line and the resultant high rate of reconnection. The ion acceleration at the shock-like structures near the separatrices controls the global ion energetics but not the rate of reconnection.

Since the ions are essentially unmagnetized in the whistler region, the electric field along the outflow direction E_x controls the ion acceleration [*Horiuchi and Sato,* 1997]. In contrast to the ions, the electrons in this region are frozen-in to the magnetic field, giving $\mathbf{E} = \mathbf{v}_e \times \mathbf{B}/c$, and thus $E_x \sim v_{ey}B_z/c$. The magnitude of B_z downstream of the x-line plays a major role in accelerating the ions away from the neutral line. Understanding how B_z scales with the parameters in the system and why it scales the way it does is therefore critical to understanding the results of the simulations. In the resistive MHD model a macroscopic current layer forms in which B_z is very small [*Biskamp,* 1986]. As a consequence the outflow acceleration of the ions is weak and the ions do not reach the Alfvén speed until they are a macroscopic distance from the x-line. This very slow acceleration causes a bottleneck of ions in the dissipation region, greatly reducing the reconnection rate.

In the hybrid simulations, however, B_z rises rapidly to a nearly constant value away from the x-line, as can be seen in the solid line in Fig. 5, which is a cut of B_z along x through the x-line from a run with $L_x = 102.4$ and $\delta_e = 0.2$. This rapid rise in the magnitude of B_z must be a consequence of the whistler physics which we demonstrate with the dashed line in Fig. 5, which is the corresponding plot of B_z from an identical simulation but with the ion motion switched off (the electron MHD limit where whistlers rather than Alfvén waves govern the dynamics). Again, B_z quickly rises to a nearly constant value over a very short distance. The rapid rise of B_z is linked to the corresponding increase in the angle θ between the separatrices. The essential physics is that the high peak whistler phase velocity prevents the electron current layer from controlling reconnection, allowing the whistler waves time to relax the stress in the newly reconnected field lines, i.e., by opening up the angle θ [*Shay and Drake,* 1998]. To quantify this statement, we argue that in a scaling sense $B_z \sim B_x$. In other words, as the parameters in the system, $c/\omega_{pe}, L,$ *etc.*, are varied, the ratio of B_z to B_x remains invariant. In the whistler region any difference in scaling between B_z and B_x must arise from a spatial anisotropy due to the presence of some small parameter in the system. In the MHD system, for example, the small width of the current layer allows $\partial/\partial z \gg \partial/\partial x$ and results in $B_x \gg B_z$ over a large region. In the whistler dominated region (outside of the electron inertial region), however, the parameter c/ω_{pe} has no influence over the reconnection rate and therefore the structure of the ion acceleration region near the x-line [*Biskamp et al.,* 1997; *Shay et al.,* 1998]. There is no other small parameter, the scale length c/ω_{pi} being absent from the pure electron system, so that in the ion acceleration zone, $\partial/\partial z \sim \partial/\partial x$, which implies that $B_z \sim B_x$.

In the whistler region the magnetic field does not remain planer. The out-of-plane electron flows, which are frozen into the kinked magnetic field, rotate the field out of the plane, turning B_x into B_y [*Mandt et al.,* 1994]. As in a whistler, $B_x \sim B_y$ and not surprisingly the reconnection electric field E_y rotates into the x direction, yielding $E_x \sim E_y$. To demonstrate these relations, we write down the massless whistler equations as $\partial\mathbf{B}/\partial t = -c\nabla \times \mathbf{E}$, with $\mathbf{E} = \mathbf{J} \times \mathbf{B}/nec$. The y component of this equation yields $\partial B_y/\partial t = -\mathbf{B}\cdot\nabla J_y/ne \sim B_x/\tau_w$ with $\tau_w = \Delta^2/\delta_i^2\Omega_{ix}$ being the whistler time based on the characteristic scale length Δ in the x and z directions. This source for B_y acts for a time $\Delta/v_x \sim \tau_w B_x/B_y$, where

$$v_x \sim J_x/ne \sim cB_y/(4\pi ne\Delta). \tag{3}$$

The various constants cancel, yielding the expected relation $B_x \sim B_y$. Similar scaling relations yield

$$E_x \sim E_y \sim B^2/(4\pi ne\Delta), \tag{4}$$

where there is now no distinction made between the various components of $\mathbf{B}$.

With the scaling of the fields in the whistler domain now understood, we can now proceed to calculate the ion acceleration near the x-line and the reconnection rate. The ions are accelerated away from the x-line by the electric field E_x until they become magnetized. In the normal situation, where the outflow velocity is comparable to or exceeds the ion thermal velocity, the magnetization location Δ is simply given by the effective Larmor radius based on the $\mathbf{E} \times \mathbf{B}$ velocity, $\Delta \sim cE_x/B\Omega_i$ [*Dungey* , 1988]. Calculating the ion velocity due to the acceleration by E_x acting over the distance Δ yields explicit expressions for the outflow velocity v_o and Δ,

$$v_o \sim c_A \quad \text{and} \quad \Delta \sim \delta_i. \tag{5}$$

Thus, the whistler induced electric field accelerates the ions to the Alfvén velocity over a space scale of the order of δ_i. The ion continuity equation in (2) then yields the result that the inflow velocity is also Alfvénic, independent of the system size! Finally, the expression in (3) for the electron outflow velocity can be re-expressed as $v_x \sim \delta_i^2\Omega_i/\Delta$. Thus, the electron velocity scales inversely with the distance from the x-line, roughly consistent with Fig. 4. At the ion magnetization point $\Delta = \delta_i$, the electron velocity also equals the Alfvén velocity, which must be the case since at this location both the ions and the electrons are frozen-in to the magnetic field and therefore must have the same velocity.

Conclusion

The results presented demonstrate clearly that the reconnection rate in a collisionless system is Alfvénic and independent of system size. Due to the presence of whistler physics in the vicinity of the x-line, the length of the ion-inertial region, or dissipation region, becomes microscopic compared to the system size for very large systems. Away from the x-line the reconnection configuration displays similarities to the Petschek model [*Petschek,* 1964] in that slow shock-like structures at least partially turn incoming ions in the outflow direction. The possibility of Alfvénic reconnection in large, collisionless systems has broad ramifications, affecting such areas as dayside magnetopause reconnection, substorm evolution in the tail, and solar flare and CME generation in the solar corona.

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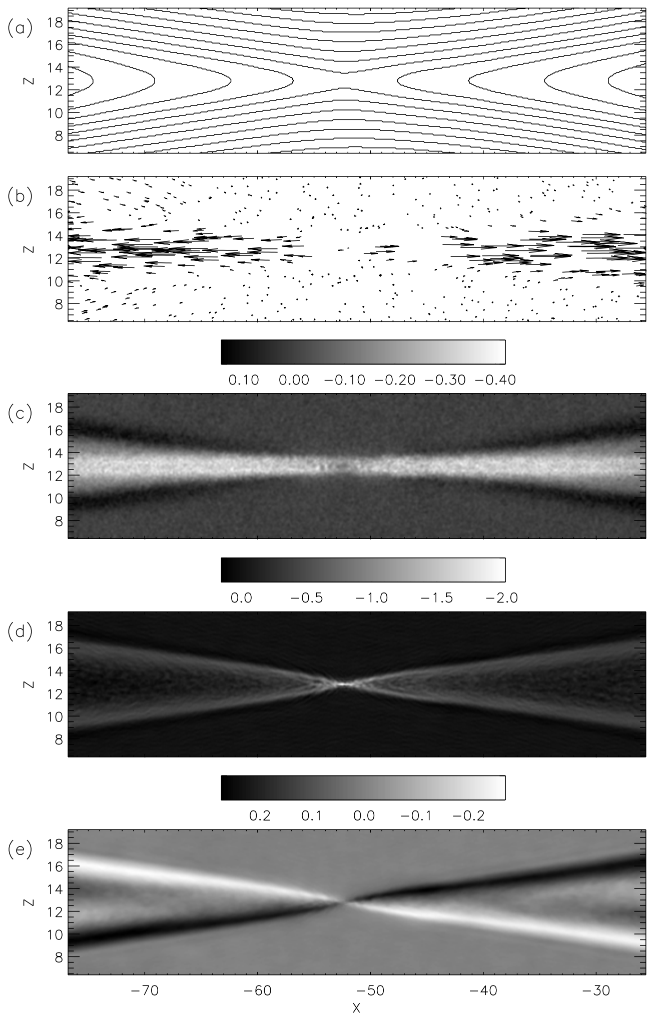

Figure 1. Structure of the x-line: (a) in-plane magnetic field, (b) in-plane ion velocity, (c) out-of-plane ion current, (d) out-of-plane electron current, (e) out-of-plane magnetic field.

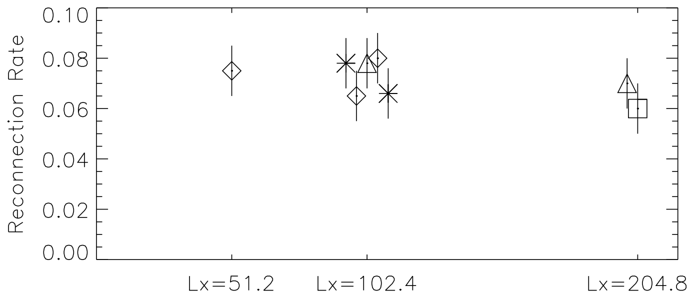

Figure 2. Reconnection rates versus L_x.

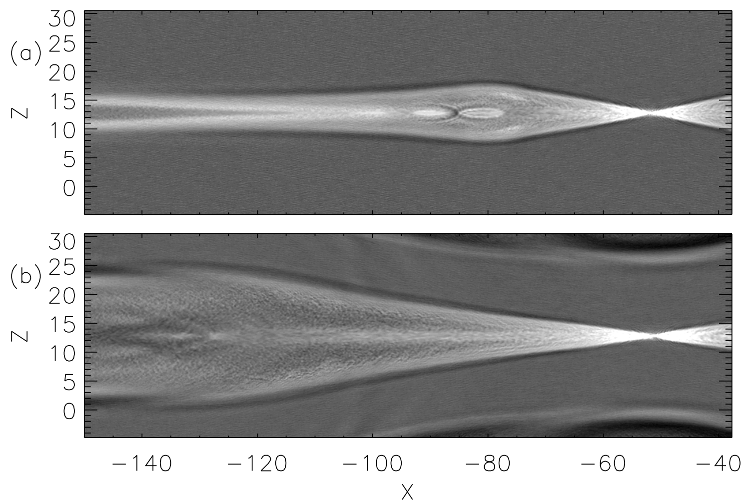

Figure 3. The out of plane current wedge at two different times.

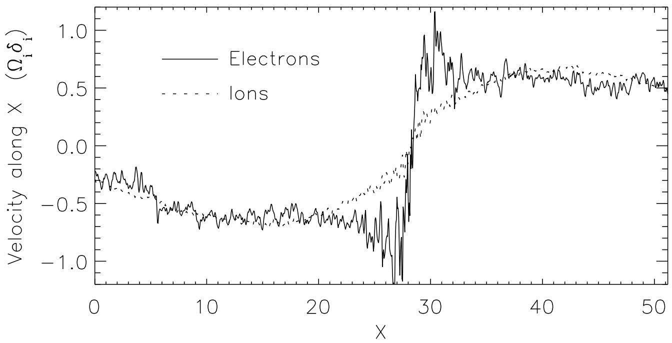

Figure 4. A cut along the outflow direction of the electron and ion outflow velocities. The x-line is at $x = 28$.

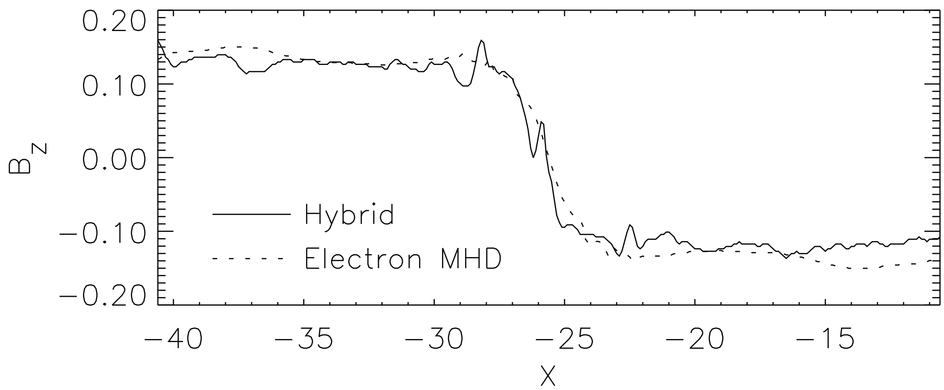

Figure 5. B_z along the outflow direction for a hybrid run and an electron MHD run. The x-line is at $x = -26$.