1. Consider the motion of a particle in a low-frequency electric field which is perpendicular to a uniform magnetic field. The equation of motion is given by

\[ \ddot{\xi} + \xi = \alpha \cos(\xi - \nu \tau) \]

where time has been normalized to the cyclotron frequency and space to the wavelength and \( \nu \ll 1 \). Because \( \nu \) is small, one might have thought that the particle motion would remain regular. However, this is not the case. The goal of this problem is to understand why.

(a) First consider the limit \( \nu = 0 \). Derive the energy invariant in this limit. What is the effective potential?

(b) Plot the contours of constant energy in the \( \dot{\xi}, \xi \) plane for \( \alpha = 0.5 \) and \( \alpha = 10\pi \). Describe the particle motion (for \( \nu = 0 \)).

(c) Derive an equation for the stationary points \( \xi_s \) of the potential. For large \( \alpha \) show the graphical solution of this equation (Plot the two terms in the equation as a function of \( \xi \). The intersections of the plots are the fixed points). Again for large \( \alpha \) what is the maximum value of \( \xi \) for which there can be a stationary point (give an approximate analytic expression) and what is the bounce frequency of the particle motion in the potential wells. Show that for large \( \alpha \) that the bounce frequency in the large wells is the same as for an electrostatic wave without a magnetic field.

(d) Now consider what happens if \( \nu \neq 0 \). Describe the behavior of the contours of constant energy (as defined in (a)) for small \( \nu \).

(e) For small \( \nu \) there exists an adiabatic invariant in the motion of some of the particles. What is this adiabatic invariant? When does the adiabatic invariant break down? Hint: recall the discussion of adiabatic invariants in Physics 761.

(f) Again considering the large \( \alpha \) limit use an argument based on the adiabatic invariant, to describe what will happen to a particle initially trapped in one of the wells at low energy. Does the adiabatic invariant remain valid for all time? What is an upper limit on the energy which can be obtained by a particle?

(g) Is the particle heating due to the action of the wave reversible? Why?