Physics 762 Homework #2 Spring '24

Dr. Drake

1. Show that the Z function satisfies the following differential equation:

$$\frac{dZ(\xi)}{d\xi} \equiv Z' = -2(1 + \xi Z(\xi)). \tag{1}$$

where $Z(\xi)$ is given by

$$Z(\xi) = \int_{-\infty}^{+\infty} \frac{ds}{\sqrt{\pi}} \frac{e^{-s^2}}{s - \xi}$$

with $Im\xi > 0$.

2. Calculate the kinetic dispersion relation for ion acoustic waves in a plasma with a Maxwellian distribution of ions. The electrons are also Maxwellian but with a mean velocity $\mathbf{U}_e = U\hat{z}$. Take the wavevector parallel to $\mathbf{U}_e$. Show that the general dispersion relation is given by

$$\epsilon(k, \omega) = 1 - \frac{k_{Di}^2}{2k^2} Z'(\frac{\omega}{kv_{ti}}) - \frac{k_{De}^2}{2k^2} Z'(\frac{\omega - kU}{kv_{te}}). \tag{2}$$

3. Show that the real frequency ω_r for sound waves when $T_e \gg T_i$ and $U \ll v_{te}$ is given by

$$\omega_r^2 = k^2 c_s^2 \frac{1}{1 + k^2/k_{De}^2}, \tag{3}$$

where $c_s^2 = T_e/m_i$. Keep corrections to obtain the growth rate of the instability,

$$\gamma = -\sqrt{\pi} \frac{\omega_r^3}{2\omega_{pi}^2} \frac{k_{De}^2}{k^2} (\frac{\omega_r - kU}{kv_{te}} + \frac{T_e}{T_i} \frac{\omega_r}{kv_{ti}} e^{-\omega_r^2/k^2 v_{ti}^2}). \tag{4}$$

For $T_i = 0$ show that instability occurs for

$$U^2(1 + k^2/k_{De}^2) > c_s^2. \tag{5}$$

Hint: you will find that $\omega \gg kv_{ti}$ and $\omega - kU \ll kv_{te}$.