1. Consider the equations for a nonlinear sound wave in a plasma with no magnetic field. Take the limit in which \( k \ll k_D \) so that dispersive terms can be neglected but include a constant viscosity term in the momentum equation. Take the system to be isothermal.

(a) Normalize the equations to get rid of all constants.

   Hint: First calculate the linear dispersion relation to determine the scaling with the wavevector \( k \).

(b) Transform to a frame of reference moving with the sound speed and develop an ordering in which dissipation balances wave steepening and the nonlinearity is weak.

(c) Using this ordering derive a nonlinear equation for the system. This is Burgers equation and is given as follows:

\[
\frac{\partial n}{\partial t} + n\frac{\partial n}{\partial x} - \frac{1}{2}\frac{\partial^2 n}{\partial x^2} = 0
\]  

(1)

(d) Assume that this equation has a steady state solution which propagates at some velocity \( v_0 \). Derive this solution and show that it takes the form of a shock. Find a relationship between the velocity of the shock and the density on either side of the shock.

   Hint: Write the solution in terms of the densities, \( n_+ \) and \( n_- \), on either side the the shock.

(e) Solve explicitly for the width of the shock in dimensional units.