Physics 606 | Final Exam | Spring 1997
Dr. Drake

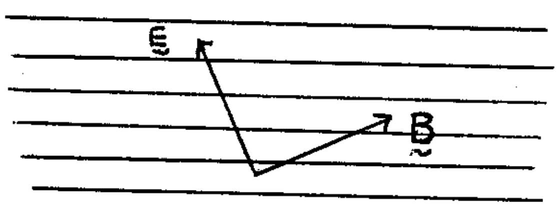
1. (70 points) The following are short answer questions which should not require extensive calculations.

   (a) Consider an infinite dielectric rod aligned with the z direction and coated with a uniform surface charge density σ. The rod oscillates around its axis with an angular displacement ϕ_0. Indicate the directions of the oscillatory components of the electric and magnetic fields. Sketch **E** in the $x-y$ plane and indicate the direction of **B**. What are the relative magnitudes of the oscillatory components of **E** and **B** at large distances from the rod. How do they fall off with distance ρ from the rod in this region? How do the fields scale with the surface charge density σ and oscillation amplitude ϕ_0?

   (b) A plane electromagnetic wave of frequency ω is normally incident on a plane conductor which has a high conductivity σ. Estimate the fraction of energy which is dissipated in the conductor during the reflection of the wave. You can simply write down the answer without doing a calculation. What happens to the energy?

   (c) Consider a very long cylindrical rod which has a permanent magnetization M oriented along its axis. The cross-sectional area of the rod is A. What is the magnetic field inside of the rod? Two such rods are placed end to end with their magnetic moments in the same direction and separated by a short distance d with $d^2 << A$. What is the magnetic field inside of the gap. Estimate the force of attraction/repulsion between the two rods. Do they attract or repell each other?

   (d) An electromagnetic wave is incident on a grid of fine, very high conductivity wires as shown. The wavelength of the light is much greater than the spacing of the wires. What happens to the wave?



2. (60 points) Consider a waveguide consisting of a vacuum enclosed by a cylindrical conductor of radius "a" and infinite extent along z. Assume that the conductivity of the metal wall is infinite. The guide is excited with an antenna of frequency ω. In the following questions relate to the lowest order TM ($B_z = 0$) mode of the guide.

   (a) Sketch the electric and magnetic field lines for this mode. What are the nonzero components of **E** and **B**?

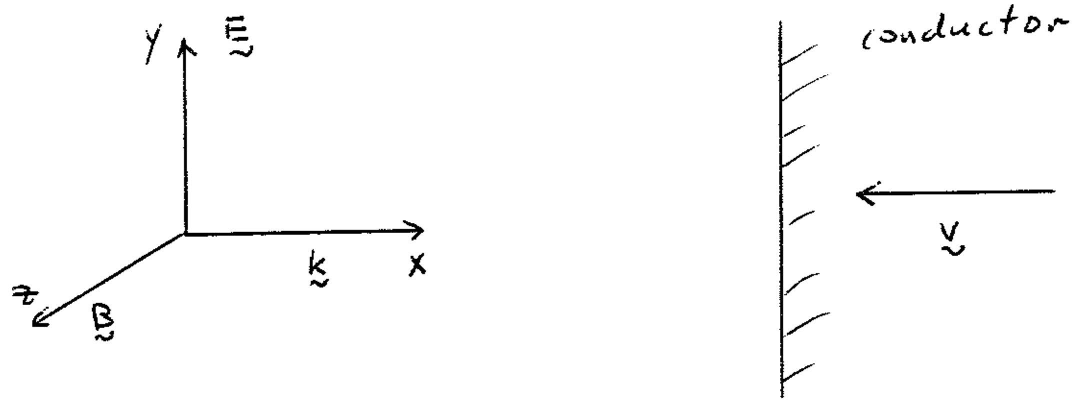
(b) Starting with Maxwell Equations derive an equation for the mode. What boundary conditions must be applied at the conducting surface?

(c) Solve the equation derived in (b) and calculate the velocity at which energy propagates down the guide. What is the limiting form of the velocity when ω is large? Small? Interpret both of these limits. What is the lowest frequency for which energy propagates down the waveguide?

3. (70 points) Consider a plane wave of frequency ω propagating along the x direction in the laboratory reference frame,

$$\mathbf{E} = E_0 \hat{y} cos(kx - \omega t).$$

An ideal plane conductor moves to the left in the laboratory frame with a velocity $\mathbf{v} = -v\hat{x}$ which may be comparable to the velocity of light.



(a) Transform the space/time dependence of the wave in S to the space/time coordinates of the frame S' moving with the conductor (assume $\mathbf{x} = \mathbf{x}'$ at $t = t' = 0$). Note that the sign of $\mathbf{v}$ is reversed from our usual convention. From the form of the wave in the S' frame, define the local wavevector k' and frequency ω' in the S' frame. How do k and ω transform under a Lorentz transformation? How does the phase of the wave $kx - \omega t$ transform? Why?

(b) Calculate the field of the right propagating wave in the S' frame.

(c) The wave reflects from the ideal conductor. Evaluate the reflected wave in the S' frame and then transform the reflected wave back to the S frame. What happens to the wave under reflection?

(d) Calculate the force per unit area on the conductor as a result of the reflection.