

1. (50 points) Consider an infinite, hollow cylindrical conductor of radius "a" which is sliced at $z = \pm L$. The segment $|z| < L$ is maintained at a potential V . The remainder of the cylinder is grounded. In the following consider only the region with $\rho < a$. Laplace's equation in cylindrical coordinates is

$$\nabla^2\Phi = \frac{1}{\rho} \frac{\partial}{\partial\rho} \rho \frac{\partial\Phi}{\partial\rho} + \frac{1}{\rho^2} \frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} = 0.$$

- (a) Sketch the electric field E in the vicinity of the ring of potential V .
- (b) Consider the limit in which $a \ll L$. Sketch the electric field in the region $\rho < a$ in this limit and sketch E_z as a function of z for $\rho = 0$. What is the scale length over which E_z becomes small outside of the region of potential V . Estimate the maximum value of E_z for $\rho = 0$.
- (c) Derive an exact expression for the potential Φ in the region $\rho < a$ for arbitrary L/a . Your answer will consist of an integral over a wavevector k . You do not need to evaluate this integral.

Hint: See the various equations at the end of the exam.

2. (50 points) Consider a hollow cylindrical grounded conductor of radius a . A charge q is positioned at the center of the conductor at $z = 0$. Assume that the charge is constrained to remain in this location. The Poisson equation for the potential inside of the cylinder is given by

$$\nabla^2\Phi = -\frac{\rho}{\epsilon_0}$$

where ρ is the charge density.

- (a) Write an expression for the charge density ρ associated with q in cylindrical coordinates.
- (b) Evaluate the potential Φ inside of the cylinder in the region $\rho < a$. What is the behavior of the potential for large z ?
Hint: see the various equations at the end of the exam.
- (c) A dielectric with $\epsilon > \epsilon_0$ is added to the region inside of the cavity where $z > z_0 > 0$. What is the direction of the force on the particle before the dielectric is inserted? After? Why?

- (d) The potential inside of the cavity is observed to change by an amount $\delta\Phi(\rho, z)$ when the dielectric was added. Write an explicit expression for the function form of this potential change for all values of z . Is this potential continuous across $z = 0$? Across $z = z_0$? Outline how you would go about obtaining an expression for the potential $\delta\Phi(\rho, z)$. You do not have to do the calculation.
- (e) Once you obtained the correction $\delta\Phi(\rho, z)$, how would you calculate the force on the charge q ?

Bessel's equation is given by

$$\rho \frac{d}{d\rho} \rho \frac{dJ_\nu(k\rho)}{d\rho} + (k^2 \rho^2 - \nu^2) J_\nu(k\rho) = 0$$

and its normalization integral is given by

$$\int_0^a d\rho \rho J_\nu^2(x_{\nu n} \rho/a) = \frac{a^2}{2} J_{\nu+1}^2(x_{\nu n}).$$

The modified Bessel equation is given by

$$\rho \frac{d}{d\rho} \rho \frac{dQ_\nu(k\rho)}{d\rho} - (k^2 \rho^2 + \nu^2) Q_\nu(k\rho) = 0$$