Equation Sheet Physics 606

Maxwell's Equations

$$\nabla \times \underset{\sim}{H} = \underset{\sim}{J} + \frac{\partial}{\partial t}\underset{\sim}{D} \qquad \underset{\sim}{D} = \epsilon \underset{\sim}{E} = \epsilon_0 \underset{\sim}{E} + \underset{\sim}{P}$$

$$\nabla \times \underset{\sim}{E} + \frac{\partial}{\partial t}\underset{\sim}{B} = 0 \qquad \underset{\sim}{B} = \mu_0(\underset{\sim}{H} + \underset{\sim}{M}) = \mu \underset{\sim}{H}$$

$$\nabla \cdot \underset{\sim}{B} = 0 \qquad \underset{\sim}{B} = \nabla \times \underset{\sim}{A}, \quad \underset{\sim}{E} = -\frac{\partial}{\partial t}\underset{\sim}{A} - \nabla\varphi$$

$$\nabla \cdot \underset{\sim}{D} = \rho$$

Math eqns

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 G(\underset{\sim}{x}, \underset{\sim}{x}') = -4\pi\,\delta(\underset{\sim}{x} - \underset{\sim}{x}')$$

$$G(\underset{\sim}{x}, \underset{\sim}{x}') = \frac{1}{|\underset{\sim}{x} - \underset{\sim}{x}'|}$$

Bessel Eqn: $$\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} + \left(1 - \frac{\nu^2}{x^2}\right) = 0$$

$$J_\nu(x) \sim x^\nu \,(\text{small } x) \sim \frac{1}{x^{1/2}}\cos\left(x - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)(\text{large } x)$$

$$N_\nu(x) \sim x^{-\nu}\,(\text{small } x) \sim \frac{1}{x^{1/2}}\sin\left(x - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)(\text{large } x)$$

$$\sim \ln x \; (\nu = 0)$$

$$H_\nu^{(1)}(x) = J_\nu + iN_\nu, \quad H_\nu^{(2)}(x) = J_\nu - iN_\nu$$

Mod Bessel Eqn: $$\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \left(1 + \frac{\nu^2}{x^2}\right) = 0$$

$$I_\nu(x) \sim x^\nu\,(\text{small } x) \sim \frac{1}{x^{1/2}}e^{x}\,(\text{large } x)$$

$$K_\nu(x) \sim x^{-\nu}\,(\text{small } x) \sim \frac{1}{x^{1/2}}e^{-x}\,(\text{large } x)$$

$$\sim \ln x\;(\nu = 0)$$

$$\int_0^a d\rho\,\rho\, J_\nu^2\left(x_{\nu n}\frac{\rho}{a}\right) = \frac{a^2}{2}J_{\nu+1}^2(x_{\nu n})$$

Legendre Eqn: $$\frac{d}{dx}(1-x^2)\frac{d}{dx} + \left[\ell(\ell+1) - \frac{m^2}{1-x^2}\right] = 0$$

$\int d\Omega\, |Y_{lm}|^2 = 1$, $Y_{lm} = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}\; P_l^m(\cos\theta)\, e^{im\varphi}$

Waves

$\mathbf{S} = \mathbf{E}\times\mathbf{H}$

$\Box^2 G = \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G = -4\pi\delta(\mathbf{x}-\mathbf{x}') \otimes \delta(t-t')$

$G = \frac{1}{|\mathbf{x}-\mathbf{x}'|}\,\delta\!\left(t' - \left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)\right)$

$\mathbf{P} = \frac{1}{c^2}\mathbf{E}\times\mathbf{H}$

wave eqn: $\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix} = 0$

Electrostatics

charge: $\Phi(\mathbf{x}) = \frac{q}{4\pi\epsilon_0|\mathbf{x}|}$ $\mathbf{F} = q\mathbf{E}$ $u_E = \frac{1}{2}\epsilon E^2$

$\mathbf{E} = -\nabla\Phi$

dipole: $\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\mathbf{x}}{|\mathbf{x}|^3}$ $\mathbf{p} = \int d\mathbf{x}\,\rho(\mathbf{x})\,\mathbf{x}$

$U_q = q\Phi$, $U_p = -\mathbf{p}\cdot\mathbf{E}$

magnetostatics

$d\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{d\mathbf{l}\times\mathbf{x}}{|\mathbf{x}|^3}$ wire: $B = \frac{\mu_0 I}{2\pi\rho}$

$\mathbf{B} = \nabla\times\mathbf{A}$

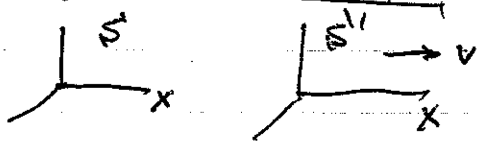
$d\mathbf{F} = I\, d\mathbf{l}\times\mathbf{B}$. $\mathbf{A} = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{x}}{|\mathbf{x}|^3}$ mag. dipole

$U_m = -\mathbf{m}\cdot\mathbf{B}$ $\mathbf{m} = \frac{I}{2}\oint d\mathbf{l}\times\mathbf{x}$

$u_B = \frac{1}{2\mu_0}B^2$ $m = IA$

Special Relativity

$\beta = v/c$ $\gamma = 1/(1-\beta^2)^{\frac{1}{2}}$



$$\begin{pmatrix} x^{0'}\\ x^{1'}\\ x^{2'}\\ x^{3'}\end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0\\ -\beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix} x^0\\ x^1\\ x^2\\ x^3\end{pmatrix}$$

4-vectors: $(ct, \mathbf{x})$, $\left(\frac{\mathcal{E}}{c}, \mathbf{p}\right)$, $\left(\frac{\omega}{c}, \mathbf{k}\right)$, $(c\rho, \mathbf{J})$

$\left(\frac{\Phi}{c^2}, \mathbf{A}\right)$ $\mathcal{E} = \gamma m c^2$, $\mathbf{p} = \gamma m \mathbf{u}$

velocity: $u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}$, $\mathbf{u}_\perp = \frac{\mathbf{u}_\perp}{\gamma_v\left(1 + \frac{v u_x'}{c^2}\right)}$

fields: $E_\parallel' = E_\parallel$, $B_\parallel' = B_\parallel$

$\frac{1}{c}\mathbf{E}_\perp' = \gamma\left(\frac{1}{c}\mathbf{E}_\perp + \boldsymbol{\beta}\times\mathbf{B}_\perp\right)$, $\mathbf{B}_\perp' = \gamma\left(\mathbf{B}_\perp - \boldsymbol{\beta}\times\mathbf{E}_\perp\frac{1}{c}\right)$