Physics 604 Homework #12 Fall '11

Dr. Drake

1. The temperature in a one-dimensional medium satisfies a diffusion equation

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2}{\partial x^2} T = 0$$

with diffusion coefficient κ over the interval (a, b), where $T(a,t) = T(b,t) = T_0$. At $t = 0$, $T(x, 0) = 0$ for $x \in (a, b)$. Solve for $T(x,t)$ by expanding in a series of sin() and/or cos() functions. At late time what is the approximate (non-trivial) time dependence?

Hint: Choose your basis functions to match your B.C.'s.

2. Consider a solid sphere of radius "a" in a world with *four spatial dimensions*. At $t = 0$ the sphere, with initial temperature of T_0 is immersed in a heat bath with $T = 0$. The temperature inside the sphere satisfies a diffusion equation.

$$\frac{\partial T}{\partial t} - \kappa \, \nabla^2 T = 0$$

where

$$\nabla^2 = \frac{1}{r^3} \frac{\partial}{\partial r} r^3 \frac{\partial}{\partial r}.$$

(a) Estimate how long it will take for the temperature of the center of the sphere to change significantly.

(b) Construct a set of basis functions $\Phi_n(r)$ such that $\Phi_n(a) = 0$ which are appropriate for solving for $T(r,t)$. Write the basis functions as a linear combination of the two solutions of Bessel's equation $Y_\nu(kr)$ and $J_\nu(kr)$. Write down expressions for the eigenvalues of your basis functions and normalize $\Phi_n(r)$ so that

$$\int_0^a dr \; w \; \Phi_n^2(r) = 1.$$

You do not have to prove that the eigenfunctions are orthogonal, but state why you are confident that they are. Sketch the lowest three eigenfunctions.

(c) Write the space/time dependence of T as

$$T(r,t) = \sum_{n=1}^{\infty} c_n(t) \Phi_n(r)$$

and solve for $c_n(t)$. At late time find an approximate expression for $T(r,t)$. How does T decay at late time?