

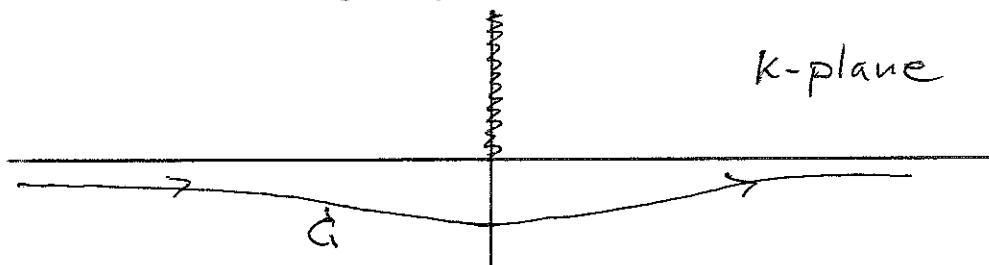
1. (a) (20 points) Evaluate

$$\int_{-\infty}^{\infty} dz \frac{(1 - \cos z)}{z^2}$$

- (b) (20 points) Evaluate

$$\int_C dk \frac{k^{1/2}}{k^2 + a^2}$$

with the contour C given by

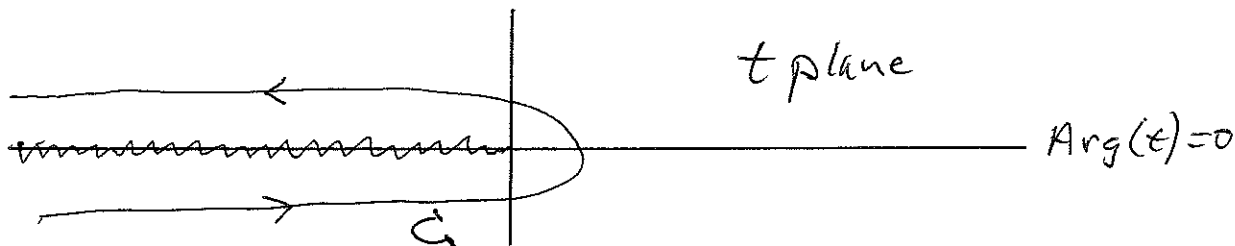


where the intercept of C with the negative imaginary axis is $-ib$ with a, b real and positive. Consider cases with $a > b$ and $b > a$.

2. (30 points) Consider the following integral

$$Q_\nu(x) = \int_C t^\nu e^{xt} dt$$

with ν real and positive and the contour C given by



- (a) For what values of complex x is the integral defined?
 (b) Evaluate $Arg(t^\nu)$ above and below the cut.
 (c) For ν real and positive express $Q_\nu(x)$ in terms of the Gamma function,

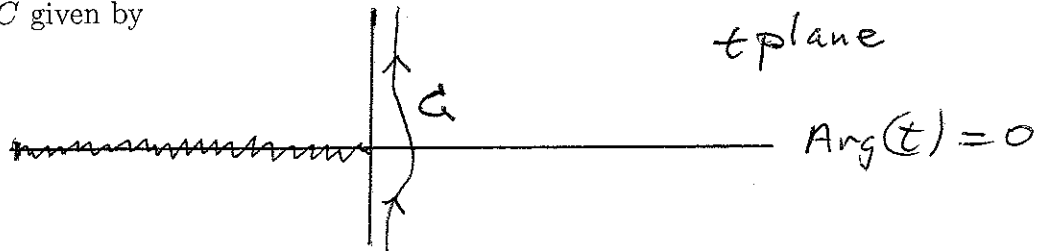
$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt.$$

- (d) For what values of complex x is the result in (c) valid?

3. (30 points) An integral representation for the parabolic cylinder function is

$$D_\nu(x) = \frac{e^{x^2/4}}{i\sqrt{2\pi}} \int_C t^\nu e^{-xt} e^{t^2/2} dt$$

with C given by



- For x large, real and positive and with $\nu \sim 1$ indicate the topology of the integrand in the t plane, locate the position of the saddle points and the PSD.
- Deform the contour to take advantage of the large parameter.
- Evaluate the integral for x large and positive.