Physics 270 Midterm #3 Spring '08
Dr. Drake

The exam consists of three problems. Please show all work and give explanations for all answers since the reasoning behind your answer is as important as the final answer itself.

1. (35 points)

   (a) (10 points) A CO_2 laser used in industrial applications produces $1.0kw$ of light at a wavelength of $10.6\mu m$. How many photons per second are produce by this laser?

   (b) (10 points) An electron in the ground state of hydrogen absorbs a photon of energy $12.75eV$. What happens to the electron? Later three photons are emitted by the same atom. What are the energies of these photons?

   (c) (10 points) The work function of gold is $5.1eV$. What is the minimum frequency f of light required to eject electrons from the surface? If the intensity of the light is doubled, by what fraction can the frequency be reduced?

   (d) (5 points) Electrons with velocity v are sent through a small hole of radius "a". Under what conditions can the wave behavior of the electrons be neglected?

2. (30 points) A scanning-tunneling-microscope (STM) is being used to measure the surface structure of a crystal.

   (a) (10 points) If the height of the potential barrier between the surface and the probe tip is $4.0eV$, what is the characteristic tunneling length η of the electrons?

   (b) (10 points) What is the probability that an electron in the crystal will reach the probe tip if the separation is $1nm$?

   (c) (10 points) As the STM scans over the surface of the material, the tunneling current measured by the STM varies by a factor of 1000. What is the height variation of the surface of the crystal?

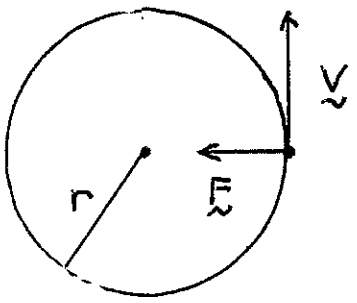
3. (35 points) Scientists recently discovered a new form of matter in the distant universe called *springium*. In this matter the force F of attraction between the electron and proton increases with the radius r so that the force on the electron is $F = -kr$. The corresponding potential energy due to this force is $kr^2/2$. The electron circles the proton in a circular orbit as in normal hydrogen.

   (a) (10 points) Calculate the velocity required for the electron to circulate in a stationary orbit of radius r as shown in the diagram.

   (b) (10 points) Calculate the allowed radii r_n of the orbits of electrons in this material.

   (c) (10 points) What are the energy levels E_n of *springium*. Express your answer in terms of $\hbar$, m_e, k and n.

   (d) (5 points) What is the energy of a photon emitted during the transition from the $n = 3$ to the $n = 1$ state. Write your answer in terms of E_1. Be sure to define E_1.



Physics 270 Exam #3 Solutions

(1) a) A CO_2 laser at 10.6 μm produces photons with energy

$$E = hf = h\frac{c}{\lambda} = \frac{6.626\times10^{-34}\ J\cdot s \times 3\times10^{8}\ m/s}{10.6\times10^{-6}\ m}$$

$$= 1.875\times10^{-20}\ J$$

$$1\ kw = 10^{3}\ \frac{J}{s}\ \frac{photon}{1.875\times10^{-20}\ J} = 5.33\times10^{22}\ \frac{photons}{s}$$

b) Electron absorbs a photon of energy 12.75eV. Since ground state is −13.6eV, its energy is −.85eV ⇒ n=4

⇒ transitions to n=4 state

Emits 3 photons: $n=4 \Rightarrow n=3$ ①

$E_n = -13.6\,ev\,\frac{1}{n^2}$ $n=3 \Rightarrow n=2$ ②

$n=2 \Rightarrow n=1$ ③

$$E_{43} = 13.6\left(\frac{1}{9}-\frac{1}{16}\right) = .661\,eV$$

$$E_{32} = 13.6\left(\frac{1}{4}-\frac{1}{9}\right) = 1.89\,eV$$

$$E_{21} = 13.6\left(1-\frac{1}{4}\right) = 10.2\,eV$$

c) Require hf = photon energy > 5.1eV

$$f > \frac{5.1\,eV}{4.14\times10^{-15}\,eV\,s} = 1.23\times10^{15}/s$$

⇒ same frequency required if intensity increases → electron absorbs a single photon

d)

Electrons are sent through a hole of radius "a". Require

$$\lambda_{DB} << a$$

to neglect wave-like behavior

$$\Rightarrow \boxed{\frac{h}{mv} << a}$$

(2)

a)

$$\eta = \frac{\hbar}{[2m(U_0 - E)]^{\frac{1}{2}}}$$

~~1.08 x 10^-34 J s~~

$$= \frac{6.58 \times 10^{-16}\ \text{eV s}\ \ 3\times10^{8}\ \text{m/s}}{(2(.511)10^{6}\ \text{eV}\ 4\text{eV})^{1/2}}$$

$$= 9.76 \times 10^{-11}\ \text{m} = \boxed{.0976\ \text{nm}}$$

b)

$$P = e^{-\frac{2w}{\eta}} = e^{-2\frac{1}{.0976}} \boxed{= 1.26 \times 10^{-9}}$$

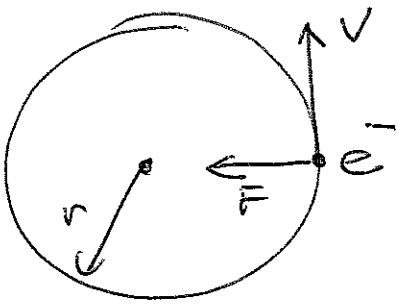
c) Let Δw = variation of the distance to the surface

$$\frac{1}{1000} = e^{-\frac{2\Delta w}{\eta}}$$

$$\frac{\Delta w}{\eta} = \frac{1}{2}\ln(1000) = 3.45$$

$$\Rightarrow \boxed{\Delta w = .337\ \text{nm}}$$

(3) a)



$$\frac{mv^2}{r} = kr \qquad v^2 = \frac{kr^2}{m}$$

$$\boxed{v = \sqrt{\frac{k}{m}}\, r} \qquad (1)$$

b) Require

$$\lambda_{DB} = \frac{h}{mv}$$

satisfy $\quad n\lambda_{DB} = 2\pi r = n\frac{h}{mv}$

$$v = n\frac{\hbar}{mr}$$

Putting this in equation (1) above yields

$$n\hbar = \sqrt{km}\, r^2$$

$$r_n = \left(\frac{n\hbar}{\sqrt{km}}\right)^{\frac{1}{2}} \qquad n = 1, 2, 3 \cdots$$

c) Energy levels?

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kr^2 = \tfrac{1}{2}m\left(\frac{k}{m}r^2\right) + \tfrac{1}{2}kr^2$$

$$= kr^2$$

$$E_n = k\frac{n\hbar}{\sqrt{km}} = \sqrt{\frac{k}{m}}\,\hbar\, n$$

d) Transition from $n=3$ to $n=1$

Let $\boxed{E_1 = \frac{1}{2}\sqrt{\frac{k}{m}}\,\hbar}$ $\qquad E_n = nE_1$

$$\Delta E = E_3 - E_1 = (3-1)E_1 = 2E_1$$

$$\boxed{\Delta E = 2E_1}$$