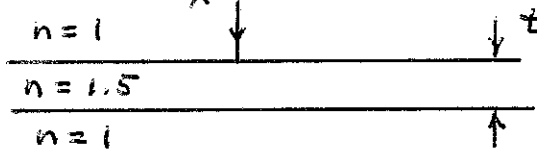


The exam consists of three problems. Please show all work and give explanations for all answers since the reasoning behind your answer is as important as the final answer itself.

1. (35 points)

(a) (5 points) Consider an electromagnetic wave of wavelength λ incident on a slit of width a . When will geometrical optics be a valid description of the passage of the light through the slit?

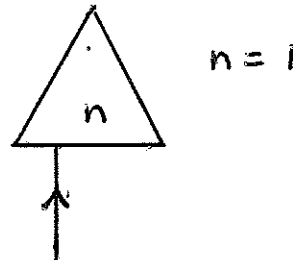
(b) (15 points) Red light of wavelength $\lambda = 650\text{nm}$ is normally incident on a slab of glass with $n = 1.5$ of thickness t as shown. What is the frequency



of the light in air ($n=1.0$)? What is the frequency f and wavelength λ of light in the glass? At what thicknesses of the glass will the reflected light be a minimum?

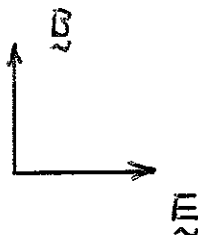
(c) (5 points) What happens to the focal length of a lens if the index of refraction of the lens material is increased?

(d) (10 points) Sketch the trajectory of a light ray passing through an equilateral triangular prism as shown. Consider two values of the index of refraction: $n = 1.1$ and $n = 3$.

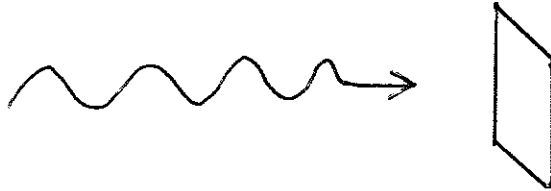


2. (30 points)

(a) (10 points) An electromagnetic wave has \mathbf{E} and \mathbf{B} in the directions shown. What is the direction of the propagation of the wave? Why?

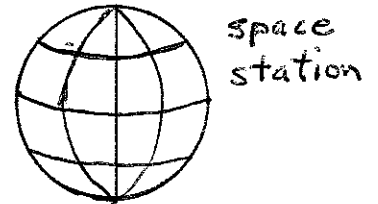
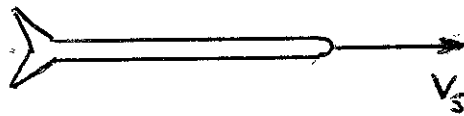


- (b) (20 points) An electromagnetic wave with Poynting flux $S = 200 \text{ kw/m}^2$ is normally incident on a flat perfectly reflecting plate of area 1000 m^2 and a mass of 0.5 kg for 100 s .
- What is the momentum density of the electromagnetic wave?
 - What is the final velocity of the plate assuming the plate has no friction with the outside world?



3. (35 points)

- (a) (10 points) A proton (rest mass 938 MeV) has a velocity of $0.99c$. What is its energy and momentum?



- (b) (25 points) An alien spacecraft attacks an Earth orbiting space station as shown above. The spacecraft is moving directly toward the space station with a velocity of $v_s = 0.5c$. The spacecraft fires a missile at the space station which has a velocity of $v_m = 0.3c$ with respect to the alien spacecraft.
- What is the velocity of the missile with respect to the space station?
 - Sensors on the space station measure the distance to the alien spacecraft as 100 km when the missile is fired. What is the distance between the space station and attacking spacecraft as seen by the alien attackers?
 - How long will it take for the missile to reach the space station as seen by the observers on the space station? As seen by the aliens?

Physics 270 Midterm #2 Solutions

①

① a) Geometrical optics is valid if $\lambda \ll a$.

$$b) f_0 \lambda_0 = c \Rightarrow f_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8 \text{ m/s}}{650 \times 10^{-9} \text{ m}}$$

$$\boxed{f_0 = 4.6 \times 10^{14} / \text{s}}$$

In the glass $\boxed{f = f_0}$

$$f \lambda = \frac{c}{n} \quad \lambda = \frac{c}{f} \frac{1}{n} = \frac{\lambda_0}{n}$$

$$\lambda = \frac{650 \text{ nm}}{1.5} = \boxed{433 \text{ nm}}$$

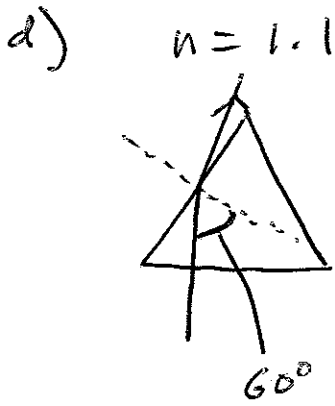
Since the light reflected from the top surface has a phase shift of π , have destructive interference when the path length down and up through the slab is a integral number of wavelengths λ

$$2t = m \lambda \Rightarrow t = m \frac{\lambda}{2}$$

$$\boxed{t = m 216 \text{ nm}}$$

c) The light hitting the lens will be bent more strongly if the index of refraction increases \Rightarrow $\boxed{f \text{ is shorter}}$

②



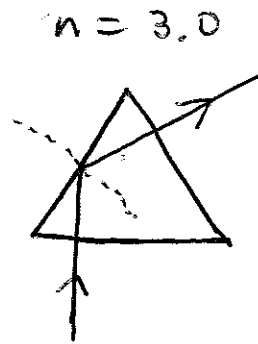
Total internal reflection for

$$\theta_i = 60^\circ > \theta_c$$

with $\sin \theta_c = \frac{1}{1.1}$

$$\theta_c = 65.4^\circ$$

\Rightarrow light transmitted



$$\sin \theta_c = \frac{1}{3} \quad \theta_c = 19.5^\circ$$

\Rightarrow light reflected

②

a)



$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

\vec{S} is out of the page

\Rightarrow direction of propagation

b)

$$i) \quad P = \frac{u}{c} = \frac{S}{c^2} = \frac{200 \times 10^3 \frac{J}{m^2 s}}{(3 \times 10^8)^2 m^2} = \frac{2}{9} \times 10^{-11} \left(\frac{kg m^2}{s^2} \right) \frac{1}{m^4}$$

$$= \boxed{2.22 \times 10^{-12} \left(\frac{kg m}{s} \right) \frac{1}{m^3}}$$

3

ii) Volume of EM rad. striking plate

$$V = 10^3 \text{ m}^2 \left(3 \times 10^8 \frac{\text{m}}{\text{s}} \right) 10^2 \cancel{\text{s}}$$

$$= 3 \times 10^{13} \text{ m}^3$$

total momentum of incident wave

$$P = 2.22 \times 10^{-12} \times 3 \times 10^{13} \frac{\text{kg m}}{\text{s}}$$

$$= 6.66 \times 10 \frac{\text{kg m}}{\text{s}}$$

$$\Delta p = \text{momentum change of plate} = 2 \times (6.66) \frac{\text{kg m}}{\text{s}}$$

\Rightarrow reflection

$$\Delta v_{\text{plate}} = \frac{2 \times 6.66}{.5} \frac{\text{m}}{\text{s}} = \boxed{26.7 \frac{\text{m}}{\text{s}}}$$

3

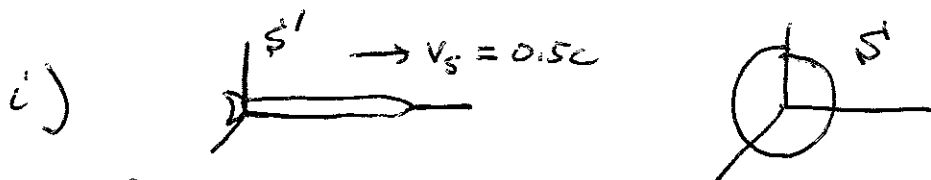
a) $v_p = .99c \Rightarrow \gamma_p = \frac{1}{\left[1 - \left(\frac{v_p}{c} \right)^2 \right]^{\frac{1}{2}}}$

$$= 7.09$$

$$E = \gamma_p m c^2 = 7.09 (938 \text{ Mev}) = \boxed{6.65 \text{ Gev}}$$

$$P = \gamma_p m v_p = \gamma_p (m c^2) \frac{v_p}{c} \frac{1}{c} = \boxed{6.58 \frac{\text{Mev}}{c}}$$

b)



missile $u_x' = .3c$

$$u_x = \frac{u_x' + v_s}{1 + \frac{v_s u_x'}{c^2}} = \frac{0.8c}{1 + .15} = \boxed{.696c}$$

④

ii) distance will appear shorter

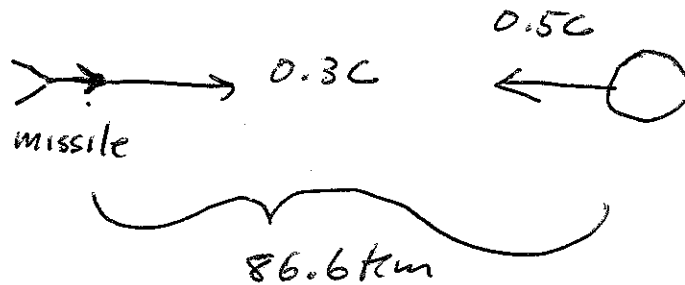
$$L = \frac{100 \text{ km}}{\gamma} = \boxed{86.6 \text{ km}}$$

$$\gamma = \frac{1}{(1 - 0.5^2)^{1/2}} = 1.15$$

ii) time as seen by space station

$$t = \frac{100 \text{ km}}{.696 \cdot 3 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{4.8 \times 10^{-4} \text{ s}}$$

According to aliens



$$t' = \frac{86.6 \text{ km}}{(.5 + .3) \cdot 3 \times 10^8 \text{ m/s}} = \boxed{3.6 \times 10^{-4} \text{ s}}$$