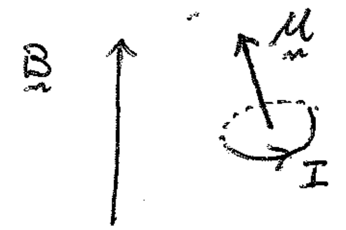
Physics 270 Midterm # 1 Solutions

Drake

(1a)



Find direction of magnetic moment μ using the right hand rule, fingers along I and thumb along μ.

Since the energy of the current loop is

$$U = -\mu \cdot B$$

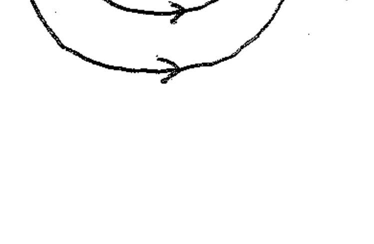
The loop will move to make this as negative as possible

$\Longrightarrow$ μ aligns with B

(b) $\oint E \cdot dl = -\frac{d}{dt} \int B \cdot dA$

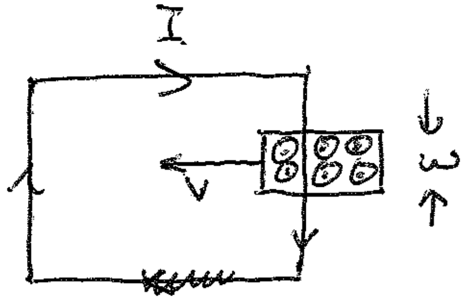
E forms loops around the solenoid

The direction is given by Lenz's law. If a conductor were present a current would flow so as to counter the change in B inside the loop



c) i) Current will flow in loop to counter the magnetic flux entering the loop $\Rightarrow$ magnetic field points into the page inside the loop.

$\Rightarrow$ clockwise current

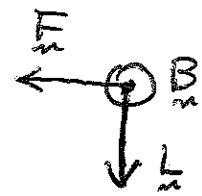


$$\frac{d\Phi}{dt} = Bwv = \mathcal{E}$$

$$\boxed{I = \frac{Bwv}{R}}$$

ii) Force on loop?

$$\underline{F} = I\,\underline{L} \times \underline{B}$$



Since current flows downward through $\underline{B}$, $\underline{L}$ is downward and force is to the left.

$$F = ILB$$

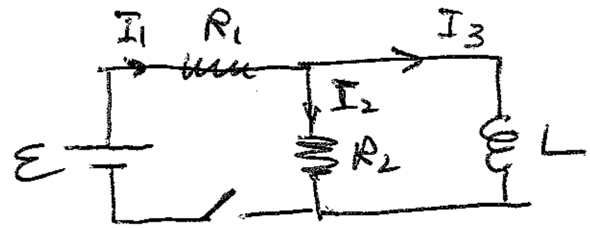
$L = w$ since only this portion of I is in a magnetic field.

$$\Rightarrow \boxed{F = \frac{B^2 w^2 v}{R}}$$

iii) Once $\underline{B}$ is entirely inside the loop, the flux linking the loop is not changing

$\Rightarrow$ no induced emf $\Rightarrow$ $\boxed{I = 0}$

2



$R_1 = 4\Omega$
$R_2 = 2\Omega$
$\mathcal{E} = 12V$
$L = 10\,\mu H$

a) I_1, I_2, I_3 just after close switch?

$I_3 = 0$ since need time for current to build up in the inductor

Use loop rule on left loop with $I_1 = I_2$

$$\mathcal{E} - I_1(R_1 + R_2) = 0$$

$$I_1 = I_2 = \frac{\mathcal{E}}{R_1 + R_2} = 2A$$

Voltage drop across inductor is same as across R_2 (loop rule of right hand loop)

$$V_L = -I_2 R_2 = -4V$$

b) After long time $\frac{dI_3}{dt} = 0 \Rightarrow$ $\boxed{V_L = 0}$

$\Rightarrow I_2 R_2 = 0$

$\Rightarrow$ $\boxed{I_2 = 0}$

Use loop rule with large loop

$\mathcal{E} - I_1 R_1 = 0$ $\boxed{I_1 = 3A}$

$\boxed{I_3 = I_1 = 3A}$

c) Open switch. What are I_1, I_2, I_3 just after switch opens?

$$\Rightarrow I_1 = 0$$

I_3 does not change $\Rightarrow$ $\boxed{I_3 = 3A}$

$$I_2 = -3A$$

$\Rightarrow$ current must flow up through R_2.

The current through L and R_2 decreases in time as the energy stored in the inductor dissipates.

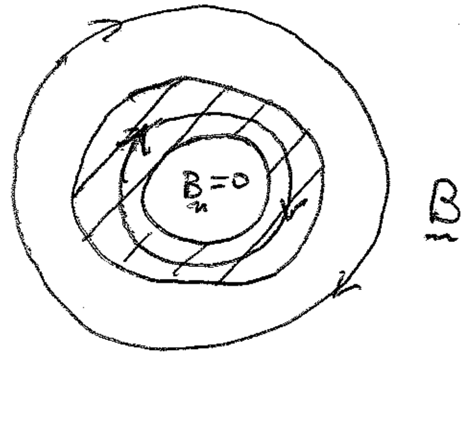
$\Rightarrow$ loop rule

$$-L\frac{dI_3}{dt} - I_3 R_2 = 0$$

$$\frac{dI_3}{dt} + \frac{R_2}{L} I_3 = 0$$

$$\tau = \frac{L}{R_2} = \frac{10\,\mu H}{2\,\Omega} = \boxed{5\,\mu s}$$

(3) a)

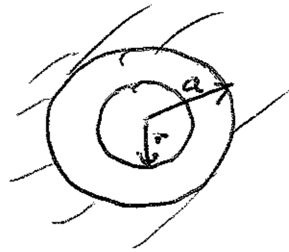


B is clockwise inside the conductor, and outside

b) $J = \dfrac{I'}{\pi(b^2-a^2)}$

c) $r < a$:

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I'$$



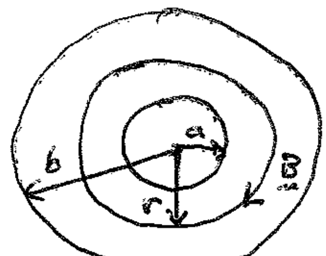
$= 0$ since no current in region $r < a$

$B\, 2\pi r = 0$

$\Rightarrow$ $\boxed{B = 0}$

$a < r < b$:

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I'$$



$B\, 2\pi r = \mu_0 J A$

$A = \pi(r^2 - a^2)$

$$B = \mu_0 \frac{I'(r^2-a^2)\pi}{\pi(b^2-a^2)\, 2\pi r}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2-a^2}{b^2-a^2}\right)}$$

$r > b$:

$$\oint \underset{\sim}{B} \cdot d\underset{\sim}{l} = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$