The exam consists of three problems. Please show all work and give explanations for all answers since the reasoning behind your answer is as important than the final answer itself.

1. (35 points)

   (a) (7 points) Consider a loop of wire with current I in an external magnetic field **B** as shown. The loop is out of the plane with the dashed portion behind and the solid in front. What is the direction of the magnetic moment of the loop? In which direction will the loop twist? Why?

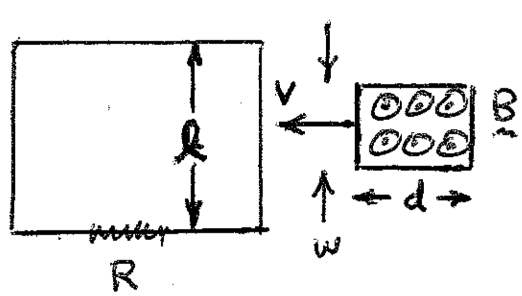
   (b) (8 points) A long solenoid produces a magnetic field pointing into the page that is increasing with time. Sketch the electric field produced by this changing magnetic field.

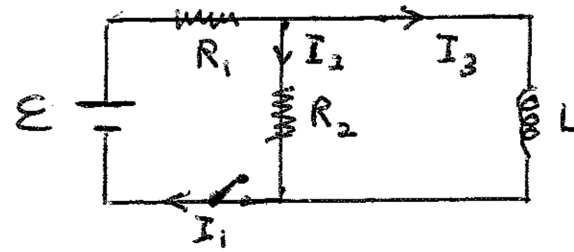
   (c) (20 points) A magnetic field **B** created by a permanent magnetic moves to the left with constant velocity v as shown and begins to cross the wire loop at $t = 0$. Assume that the motion of the loop is negligible.

      i. As the magnetic field enters the loop in which direction will the current in the loop flow? Calculate the current in the loop.

      ii. What is the direction of the force on the loop? Calculate the magnitude of the force.

      iii. Once the magnetic field lies entirely within the loop, what is the current?





2. (35 points) Consider the circuit with emf, resistors and inductor as shown. At $t = 0$ the switch is closed. Take $R_1 = 4\Omega$, $R_2 = 2\Omega$, $\varepsilon = 12v$ and $L = 10\mu H$.

   (a) Just after the switch is closed, calculate I_1, I_2 and I_3. What is the voltage drop across the inductor at this time?

   (b) After a long time has passed again calculate I_1, I_2 and I_3. What is the potential drop across the inductor at this time?

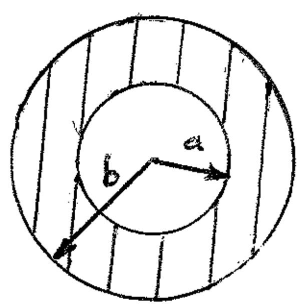
   (c) After the currents have reached steady values, the switch is opened. What are the currents I_1, I_2 and I_3 just after open switch. What happens as time passes? Write down an equation for the current I_3 through the inductor. Identify and evaluate the time constant for this equation.

3. (30 points) Consider an infinitely long cylindrical conductor with inner radius a and outer radius b, carrying a total current I going into the page. The current is uniformly distributed throughout the conductor.

   (a) Sketch the magnetic field lines.

   (b) What is the current density J in the conductor? (Hint: the cross sectional area of the conductor is $\pi(b^2 - a^2)$).

   (c) Calculate $\mathbf{B}$ in the three regions $r < a$, $a < r < b$, and $r > b$.



Formulae

Magnetostatics

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I'$$

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{d\mathbf{L} \times \hat{r}}{r^2}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} \frac{Tm}{A}$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{wire}$$

$$B = \mu_0 I n \quad \text{solenoid}$$

$$\mathbf{F} = I\, \mathbf{L} \times \mathbf{B} \quad \text{wire}$$

$$\mathbf{F} = q\, \mathbf{v} \times \mathbf{B} \quad \text{charge}$$

$$U = -\boldsymbol{\mu} \cdot \mathbf{B} \qquad \mu = IA$$

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \qquad \omega_B = \frac{qB}{m}$$

$$u = \frac{B^2}{2\mu_0}$$

Induction

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

$$\Phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

$$L = N\Phi / I$$

Misc

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

$$C = AB\sin\theta$$

areas πr^2 circle
$4\pi r^2$ sphere

Circuits

$$\sum_i I_i = 0 \quad \text{point rule}$$

$$\sum_i V_i = 0 \quad \text{loop rule}$$

$$I = \frac{dQ}{dt}$$

$$V = \frac{Q}{C}, \quad V = -IR$$

$$V = -L\frac{dI}{dt}, \quad \tau = RC, \quad \tau = \frac{L}{R}$$

$$U = \frac{1}{2}LI^2, \quad U = \frac{1}{2}CV^2$$

$$P = I^2 R$$

$$R = R_1 + R_2 + \ldots \quad \text{series}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \quad \text{parallel}$$

Circular Motion

$$a = \frac{v^2}{R} = \omega^2 R$$

$$v = \omega R$$