Physics 270, Assignment 9

38.8

We have electrons leaving a copper surface via the photoelectric effect with a maximum kinetic energy $K = 1.10 \text{ eV}$. The work function for copper is $E_0 = 4.65 \text{ eV}$. Thus, we have the energy of the photon as $E_\gamma = h\nu = \frac{hc}{\lambda}$ as

$$E_\gamma = K + E_0 = (1.10 \text{ eV}) + (4.65 \text{ eV}) = 5.75 \text{ eV}$$

Alternatively, we may write

$$5.75 \text{ eV} = \frac{hc}{\lambda}$$

Solving for the wavelength, we have

$$\lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{5.75 \text{ eV}} = 215.65 \text{ nm}$$

38.12

A photon with wavelength $\lambda = 700 \text{ nm}$ has energy

$$E_\gamma = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{700 \text{ nm}} = 1.77 \text{ eV}$$

Similarly, a $5 \text{ keV}$ x-ray photon has wavelength given by

$$5 \times 10^3 \text{ eV} = E_\gamma = \frac{hc}{\lambda}$$

The wavelength is then

$$\lambda = \frac{hc}{E_\gamma} = \frac{1240 \text{ eV} \cdot \text{nm}}{5 \times 10^3 \text{ eV}} = 0.248 \text{ nm}$$

38.16

The Debroglie wavelength of the electron is given by $\lambda = \frac{h}{p}$. For electrons with very short wavelengths, the velocity of the electron is very big. This means that we shall have to use the relativistic formula for momentum: $p = \gamma_p mv$. We may therefore write

$$\lambda = \frac{h}{p} = \frac{h}{\gamma_p mv} = \frac{h}{\gamma_p mc^2} = \frac{1}{\gamma_p mc^2} \frac{hc}{v}$$

In terms of the unitless parameter $\beta$, where $v = \beta c$, we have

$$\lambda = \sqrt{1 - \beta^2} \frac{hc}{mc^2} \frac{1}{\beta}$$

We know that the mass energy of the electron is $0.511 \text{ MeV}$. Thus we may write

$$\lambda = 10^{-12} \text{ m} = 10^3 \text{ fm} = \sqrt{1 - \beta^2} \frac{1240 \text{ MeV} \cdot \text{fm}}{0.511 \text{ MeV}}$$

or

$$0.412 = \frac{\sqrt{1 - \beta^2}}{\beta}$$

Solving for $\beta$, we get $\beta = 0.92$. This gives us a speed of $v = \beta c = 2.77 \times 10^8 \text{ m/s}$. 
part b,c,d

For the next few cases, the wavelength is large enough (meaning that \( v \) is small enough) that we may use the Newtonian approximation \( p = mv \). Thus, we have

\[
\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{hc}{mc^2 v} = \frac{hc}{mc^2} \frac{1}{\beta} = \frac{1240 \text{ MeV} \cdot \text{fm}}{0.511 \text{ MeV} \cdot \beta}
\]

Solving, for \( \beta \) in each case gives

\[
\begin{align*}
\beta_B &= \frac{1240 \text{ MeV} \cdot \text{fm}}{0.511 \text{ MeV} \cdot \lambda} = \frac{1240 \text{ MeV} \cdot \text{fm}}{0.511 \text{ MeV} \cdot 1\text{nm}} = 2.42 \times 10^{-3} \\
\beta_C &= \frac{1240 \text{ MeV} \cdot \text{fm}}{0.511 \text{ MeV} \cdot \lambda} = \frac{1240 \text{ MeV} \cdot \text{fm}}{0.511 \text{ MeV} \cdot 1\mu\text{m}} = 2.42 \times 10^{-6} \\
\beta_D &= \frac{1240 \text{ MeV} \cdot \text{fm}}{0.511 \text{ MeV} \cdot \lambda} = \frac{1240 \text{ MeV} \cdot \text{fm}}{0.511 \text{ MeV} \cdot 1\text{um}} = 2.42 \times 10^{-9}
\end{align*}
\]

Resulting in speeds,

\[
\begin{align*}
v_B &= \beta_B c = 725516 \text{ m/s} \\
v_C &= \beta_C c = 725.516 \text{ m/s} \\
v_D &= \beta_D c = 0.725516 \text{ m/s}
\end{align*}
\]

38.32

The Hydrogen transitions in question have energies

\[
\begin{align*}
\Delta E_{4\rightarrow1} &= E_4 - E_1 = \frac{-13.6 \text{ eV}}{4^2} - \frac{-13.6 \text{ eV}}{1^2} = 12.75 \text{ eV} \\
\Delta E_{4\rightarrow2} &= E_4 - E_2 = \frac{-13.6 \text{ eV}}{4^2} - \frac{-13.6 \text{ eV}}{2^2} = 2.55 \text{ eV} \\
\Delta E_{4\rightarrow3} &= E_4 - E_3 = \frac{-13.6 \text{ eV}}{4^2} - \frac{-13.6 \text{ eV}}{3^2} = 0.66 \text{ eV}
\end{align*}
\]

The wavelengths given by equation \( \Delta E = E_\gamma = \frac{hc}{\lambda} \) (which can be rearranged to give \( \lambda = \frac{hc}{\Delta E} \)) are

\[
\begin{align*}
\lambda_{4\rightarrow1} &= \frac{hc}{\Delta E_{4\rightarrow1}} = \frac{1240 \text{ eV} \cdot \text{nm}}{12.75 \text{ eV}} = 97.25 \text{ nm} \\
\lambda_{4\rightarrow2} &= \frac{hc}{\Delta E_{4\rightarrow2}} = \frac{1240 \text{ eV} \cdot \text{nm}}{2.55 \text{ eV}} = 486.27 \text{ nm} \\
\lambda_{4\rightarrow3} &= \frac{hc}{\Delta E_{4\rightarrow3}} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.66 \text{ eV}} = 1878.79 \text{ nm}
\end{align*}
\]

38.64

Now we have two Hydrogen atoms colliding head on. Their combined initial energy will be the sum of their relativistic kinetic energies (Because their speeds are assumed equal, we have \( K_1 + K_2 = 2K \) where \( K = (\gamma - 1) mc^2 \) is the kinetic energy of one of the particles). We also know that the mass energy, \( mc^2 \), of a hydrogen atom (assumed equal to that of a proton) is 938 MeV.

\[
E_i = 2K = 2(\gamma - 1) mc^2 = 2 \left( \frac{1}{\sqrt{1 - \beta^2}} - 1 \right) mc^2 = 2 \left( \frac{1}{\sqrt{1 - \beta^2}} - 1 \right) (938 \text{ MeV})
\]
After the collision, the atoms are at rest and there are two photons. Each photon has energy \( E = \frac{hc}{\lambda} \). Thus, the final energy is

\[
E_f = 2E_\gamma = 2 \frac{hc}{\lambda} = 2 \frac{1240 \text{ eV} \cdot \text{nm}}{121.6 \text{ nm}} = 20.39 \text{ eV} = 20.39 \times 10^{-6} \text{ MeV}
\]

By conservation of energy we have \( E_i = E_f \). This gives us

\[
2 \left( \frac{1}{\sqrt{1 - \beta^2}} - 1 \right) (938 \text{ MeV}) = 20.39 \times 10^{-6} \text{ MeV}
\]

or

\[
\sqrt{1 - \beta^2} = \frac{1}{1 + 1.087 \times 10^{-8}}
\]

Solving for \( \beta \) yields \( \beta = 0.000147 \) giving us a speed of

\[
v = \beta c = 4.4 \times 10^4 \text{ m/s}
\]

Notice that this speed was low enough that we probably could have gotten away with using the Newtonian approximation \( p = mv \).

### 38.62

Here we may simply use equation 38.36

\[
\lambda_{n \rightarrow m} = \frac{\lambda_0}{m^2 - \frac{1}{n^2}}
\]

and equation 38.37

\[
\lambda_0 = \frac{91.18 \text{ nm}}{Z^2}
\]

to give us (where \( Z = 2 \) for \( \text{He}^+ \))

\[
\lambda_{n \rightarrow m} = \frac{22.79 \text{ nm}}{m^2 - \frac{1}{n^2}}
\]

This will give us

\[
\begin{align*}
\lambda_{4 \rightarrow 3} &= \frac{22.79 \text{ nm}}{\frac{1}{3^2} - \frac{1}{4^2}} = 468.82 \text{ nm} \\
\lambda_{4 \rightarrow 2} &= \frac{22.79 \text{ nm}}{\frac{1}{4^2} - \frac{1}{5^2}} = 121.54 \text{ nm} \\
\lambda_{4 \rightarrow 1} &= \frac{22.79 \text{ nm}}{\frac{1}{5^2} - \frac{1}{4^2}} = 24.30 \text{ nm}
\end{align*}
\]

### 38.22

Here we have the energy spectrum of an atom such that

\[
\Delta E_{3 \rightarrow 1} = 4.0 \text{ eV}, \quad \Delta E_{3 \rightarrow 2} = 2.5 \text{ eV}, \quad \Delta E_{2 \rightarrow 1} = 1.5 \text{ eV}
\]

These result in emission photon energies (given by \( E_\gamma = h\nu = \frac{hc}{\lambda} \)) of

\[
\begin{align*}
\lambda_{3 \rightarrow 1} &= \frac{hc}{\Delta E_{3 \rightarrow 1}} = \frac{1240 \text{ eV} \cdot \text{nm}}{4.0 \text{ eV}} = 310 \text{ nm} \\
\lambda_{3 \rightarrow 2} &= \frac{hc}{\Delta E_{3 \rightarrow 2}} = \frac{1240 \text{ eV} \cdot \text{nm}}{2.5 \text{ eV}} = 496 \text{ nm} \\
\lambda_{2 \rightarrow 1} &= \frac{hc}{\Delta E_{2 \rightarrow 1}} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.5 \text{ eV}} = 826.6 \text{ nm}
\end{align*}
\]

The wavelengths for absorption are the same as those for emission: \( \lambda_{1 \rightarrow 2} = \lambda_{2 \rightarrow 1} \) and \( \lambda_{1 \rightarrow 3} = \lambda_{3 \rightarrow 1} \).
38.30
We wish to show that the first three states of hydrogen have angular momenta \( n \hbar \). We may simply follow the derivation of 38.32 to give us
\[
L = n \hbar
\]

38.71
First we want the orbital radius and speed of a muon in the \( n = 1 \) ground state of a carbon nucleus \((Z = 6)\). We may simply treat the muon as a heavy \((m_\mu = 207m_e)\) electron. As such, we have from equation 38.37
\[
r_n = \frac{n^2 a_\mu}{Z} = \frac{1}{207} \frac{n^2 a_B}{Z}
\]
where \( a_\mu = \frac{a_B}{m_e} \) (this is because of the \( m_e \) in the denominator of Bohr radius in the middle of page 1240 above equation 38.26). This gives us
\[
r_{\mu 1} = \frac{1}{207} \frac{a_B}{6} = \frac{1}{207} \frac{0.0529 \text{ nm}}{6} = 4.25 \times 10^{-5} \text{ nm}
\]
The velocity becomes
\[
v_{\mu 1} = Z v_1 = Z \frac{\hbar}{m_\mu a_\mu} = Z \frac{\hbar}{m_e a_B} = 6 \left( 2.19 \times 10^6 \text{ m/s} \right) = 1.31 \times 10^7 \text{ m/s}
\]
according to the (nonrelativistic) Bohr model.

part b, c
The energy of the \( 2 \to 1 \) transition is
\[
\Delta E = Z E_{\mu 1} \left( \frac{1}{2^2} - \frac{1}{1^2} \right)
\]
where \( E_1 \) for Hydrogen is proportion to \( \frac{1}{a_B} \). As such \( E_{\mu 1} = 207E_1 = -2815.2 \text{ eV} \). Hence, we have
\[
\Delta E = Z E_{\mu 1} \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = 6 \left( -2815.2 \text{ eV} \right) \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = 12668.4 \text{ eV}
\]
This results in a wavelength of an emitted photon of the form
\[
\lambda_{2\to 1} = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{12668.4 \text{ eV}} = 0.097 \text{ nm}
\]
which is in the X-ray range.

part d
Lastly, we would like to know how many orbits the muon will complete in a muon lifetime \( \tau = 1.5 \mu s \). This number will be the linear distance the muon travels \( v \tau \) in that time divided by the circumference \( 2\pi r \) of the orbit it makes. Hence, we have
\[
n = \frac{v_1 \tau}{2\pi r_1} = \frac{\left( 1.31 \times 10^7 \text{ m/s} \right) \left( 1.5 \times 10^{-6} \text{ s} \right)}{2\pi \left( 4.25 \times 10^{-14} \text{ m} \right)} = 7.35 \times 10^{13}
\]
which is a very large number of orbits. As such, we may be confident that we can use the Bohr model (good for bound states) to approximate the muon’s orbit.