Physics 270, Assignment 10

39.6

We know that the probability of detecting a photon on a strip is proportional to the Intensity $I = \frac{P}{A}$ which is proportional to the squar of the electric field. As such, we have

$$2000 = CE_1^2 = C\left(10\ \text{V/m}\right)^2$$

on the first strip and

$$N = C\left(30\ \text{V/m}\right)^2$$

at the other strip. Dividing the second equation by the first, we have

$$\frac{N}{2000} = \frac{30^2}{10^2}$$

or $N = 18000$ photons.

39.14

We know that the squared modulus $\left|\psi\left(x\right)\right|^2$ must normalize to 1 (the particle must be somewhere). In other words, the area under the curve $f\left(x\right) = \left|\psi\left(x\right)\right|^2$ must equal 1. Using simple geometry, we may write

$$2a = 1 \quad \text{or} \quad a = \frac{1}{2}$$

Furthermore, the probability of the electron being between $x = 1.0$ nm and $x = 2.0$ nm is equal to the area under the curve $f\left(x\right) = \left|\psi\left(x\right)\right|^2$ on this interval. We easily have.

$$P\left(1.0\ \text{nm}\ \leq x_e \leq 2.0\ \text{nm}\right) = \frac{1}{2}\left(\frac{a}{2}\right)(1) = \frac{1}{2}\left(\frac{\frac{1}{2}}{2}\right)(1) = \frac{1}{8}$$

39.44

We know that the Heisenberg uncertainty principle gives us the constraint

$$\Delta x \Delta p \geq \frac{h}{2}$$

We know that the particle is in a box of length 1mm. We therefore have $\Delta x = 1\ \text{mm} = 10^{12}$ fm and $\Delta p = m\Delta v$. We therefore have

$$m\Delta v \geq \frac{h}{2\Delta x}$$

If we consider that the mass energy of sodium is about 23 times that of hydrogen ($m_H c^2 = 938$ MeV), then this gives us a minimum range of velocities Δv as

$$\begin{aligned} \Delta v &\geq \frac{h}{2m\Delta x} = \frac{h}{2m\Delta x}\frac{c^2}{c^2} = \frac{hc}{2\left[mc^2\right]\Delta x}c = \frac{(1240\ \text{MeV}\cdot\text{fm})}{2\left[23\left(938\ \text{MeV}\right)\right]\left(10^{12}\ \text{fm}\right)}\left(3\times10^8\ \text{m/s}\right) \\ &= 8.6\times10^{-6}\ \text{m/s} \end{aligned}$$

If we now assume that the root mean square velocity of these sodium particles is half of what we found for the range of velocities, then we have

$$v_{rms} = 4.3\times10^{-6}\ \text{m/s}$$

We can then use equation 18.26

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

or

$$\begin{aligned} T &= \frac{m}{3k_B}v_{rms}^2 = \frac{m}{3k_B}v_{rms}^2\frac{c^2}{c^2} = \frac{mc^2}{3k_B}\left(\frac{v_{rms}}{c}\right)^2 = \frac{23\,(938\text{ MeV})}{3\left(8.61\times10^{-5}\text{ eV K}^{-1}\right)}\left(\frac{v_{rms}}{c}\right)^2 \\ &= \frac{23\,(938\text{ MeV})}{3\left(8.61\times10^{-11}\text{ MeV K}^{-1}\right)}\left(\frac{4.3\times10^{-6}}{3\times10^{8}}\right)^2 = 1.71\times10^{-14}\text{ K} \end{aligned}$$

40.2

Firstly, the wavelength of the released photons ($\lambda = 1484$ nm) is in the infrared spectrum. They have an energy equal to energy of the 3 → 2 transition for the particles in the box.

$$E_{3\to2} = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{1484\text{ nm}} = 0.835\text{ eV}$$

We know that the energy of the 3 → 2 transition for electrons in the box is also given by

$$E_{3\to2} = \frac{h^2}{8mL^2}\left(3^2-2^2\right) = \frac{h^2}{8mL^2}\frac{c^2}{c^2}\left(3^2-2^2\right) = \frac{(hc)^2}{8\left(mc^2\right)L^2}(5) = \frac{5\,(1240\text{ eV}\cdot\text{nm})^2}{8\,(511\times10^3\text{ eV})\,L^2}$$

By equating these two expressions, we have

$$0.835\text{ eV} = \frac{5\,(1240\text{ eV}\cdot\text{nm})^2}{8\,(511\times10^3\text{ eV})\,L^2}$$

or

$$L^2 = \frac{5\,(1240\text{ eV}\cdot\text{nm})^2}{8\,(511\times10^3\text{ eV})\,(0.835\text{ eV})} = 2.25\text{ nm}^2$$

Taking a square root of both sides yields $L = 1.5$ nm.

40.26

We shall now model an atom as an electron in a box of length $L = 0.10$ nm. The energy levels are thus given by

$$E_n = n^2\frac{h^2}{8mL^2} = n^2\frac{h^2}{8mL^2}\frac{c^2}{c^2} = n^2\frac{(hc)^2}{8\left(mc^2\right)L^2} = n^2\frac{(1240\text{ eV}\cdot\text{nm})^2}{8\,(511\times10^3\text{ eV})\,(0.10\text{ nm})^2} = n^2\,(37.61\text{ eV})$$

As such, $E_1 = 37.61$ eV, $E_2 = 150.45$ eV, $E_3 = 338.51$ eV, and $E_4 = 601.8$ eV. We shall easily calculate wavelengths of the emission spectrum from $\Delta E_{i\to j} = E_\gamma = \frac{hc}{\lambda}$. Thus, we have

$$\begin{aligned} \lambda_{2\to1} &= \frac{hc}{\Delta E_{2\to1}} = \frac{1240\text{ eV}\cdot\text{nm}}{(150.45\text{ eV}-37.61\text{ eV})} = 10.98\text{ nm} \\ \lambda_{3\to1} &= \frac{hc}{\Delta E_{3\to1}} = \frac{1240\text{ eV}\cdot\text{nm}}{(338.51\text{ eV}-37.61\text{ eV})} = 4.12\text{ nm} \\ \lambda_{3\to2} &= \frac{hc}{\Delta E_{3\to2}} = \frac{1240\text{ eV}\cdot\text{nm}}{(338.51\text{ eV}-150.45\text{ eV})} = 6.59\text{ nm} \\ \lambda_{4\to1} &= \frac{hc}{\Delta E_{4\to1}} = \frac{1240\text{ eV}\cdot\text{nm}}{(601.8\text{ eV}-37.61\text{ eV})} = 2.19\text{ nm} \\ \lambda_{4\to2} &= \frac{hc}{\Delta E_{4\to2}} = \frac{1240\text{ eV}\cdot\text{nm}}{(601.8\text{ eV}-150.45\text{ eV})} = 2.74\text{ nm} \\ \lambda_{4\to3} &= \frac{hc}{\Delta E_{4\to3}} = \frac{1240\text{ eV}\cdot\text{nm}}{(601.8\text{ eV}-338.51\text{ eV})} = 4.70\text{ nm} \end{aligned}$$

All of these wavelengths are in the ultraviolet range. The fact that the Bohr energies depends on the choice of $E = 0$. This choice is immaterial. Physics depends on energy differences; not the absolute energies themselves.

40.38

Here we want the probability for an electron of energy $E = 0$ to tunnel through an energy barrier (the work function) of $U_0 = 4.3$ eV that is 50 nm wide. This probability is given by equation 40.53.

$$P = e^{-2w/\eta}$$

where $w = 50$ nm and η (the penetration depth) has the form from equation 40.41

$$\eta = \frac{\hbar}{\sqrt{2m\left(U_0 - E\right)}} = \frac{h}{2\pi\sqrt{2mU_0}}\frac{c}{c} = \frac{hc}{2\pi\sqrt{2mc^2U_0}} = \frac{1240 \text{ eV}\cdot\text{nm}}{2\pi\sqrt{2\left(511\times10^3 \text{ eV}\right)\left(4.3 \text{ eV}\right)}} = 0.094 \text{ nm}$$

We thus have

$$P = e^{-2w/\eta} = e^{-2(50)/(0.084)} = 9.61\times10^{-518} \approx 0$$

41.54

Now we have a P =100 MW laser releasing a $\Delta t = 40.0$ ns long pulse with a wavelength of λ =690 nm. We wish to know how many stimulated emissions were necessary to create this pulse. The total energy of the pulse is

$$E = P\Delta t = \left(100\times10^6 \text{ J/s}\right)\left(40\times10^{-9} \text{ s}\right) = 4.0 \text{ J} = 2.49\times10^{19} \text{ eV}$$

We also know that each photon created by stimulated emission has energy

$$E_\gamma = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{690 \text{ nm}} = 1.79 \text{ eV}$$

The total number of photons (and therefore the total number of stimulated emissions) is

$$N = \frac{E}{E_\gamma} = \frac{2.49\times10^{19} \text{ eV}}{1.79 \text{ eV}} = 1.38\times10^{19}$$

40.22

Here we are given that a spherical droplet of 2.0 μm diameter (radius $r = 1.0$ μm) is moving with a speed of 1.0 μm/s in a box of length $L = 20$ μm. The mass of the water droplet is then

$$m = \rho V = \rho\frac{4}{3}\pi r^3 = \left(1000 \text{ kg/m}^3\right)\frac{4}{3}\pi\left(1\times10^{-6} \text{ m}\right)^3 = 4.18\times10^{-15} \text{ kg}$$

The kinetic energy can be equated to the energy state of a particle in a box

$$\frac{1}{2}mv^2 = E = n^2\frac{h^2}{8mL^2}$$

or

$$n = \sqrt{\frac{4m^2L^2v^2}{h^2}} = 2mv\frac{L}{h} = 2\left(4.18\times10^{-15} \text{ kg}\right)\left(10^{-6} \text{ m/s}\right)\frac{20\times10^{-6} \text{ m}}{6.63\times10^{-34} \text{ Js}} = 2.52\times10^8$$

This quantum number is so high we can very safely use classical mechanics approximate the evolution of this system.

40.28

We are given the energies of two adjacent energy levels for a particle in a box of length 10 fm.

$$\begin{aligned} E_n &= n^2 \frac{h^2}{8mL^2} = 32.9 \text{ MeV} \\ E_{n+1} &= (n+1)^2 \frac{h^2}{8mL^2} = 51.4 \text{ MeV} \end{aligned}$$

By dividing the second equation by the first, we easily have

$$\frac{(n+1)^2}{n^2} = \frac{51.4}{32.9}$$

Solving for n, we get $n = 4$ and $n+1 = 5$. The wavelength of the photon emitted in the $n+1 \to n$ transition has energy $\Delta E = E_\gamma = \frac{hc}{\lambda}$. This gives us

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{(51.4 \times 10^6 \text{ eV} - 32.9 \times 10^6 \text{ eV})} = 6.70 \times 10^{-5} \text{ nm}$$

To find the mass, let us begin with

$$32.9 \text{ MeV} = E_4 = 4^2 \frac{h^2}{8mL^2} = 2\frac{h^2}{mL^2}\frac{c^2}{c^2} = 2\frac{(hc)^2}{(mc^2)\,L^2} = 2\frac{(1240 \text{ MeV fm})^2}{(mc^2)\,(10 \text{ fm})^2}$$

Solving for the mass energy of the particle $E_0 = mc^2$, we have

$$E_0 = mc^2 = 2\frac{(1240 \text{ MeV fm})^2}{(32.9 \text{ MeV})\,(10 \text{ fm})^2} = 934.71 \text{ MeV}$$

This is about the mass energy of a proton or neutron. No other particle is even close. In kilograms, the mass is around

$$m_{p,n} \simeq 1.67 \times 10^{-27} \text{ kg}$$