Physics 270, Assignment 10

39.6
We know that the probability of detecting a photon on a strip is proportional to the Intensity $I = \frac{P}{A}$ which is proportional to the square of the electric field. As such, we have

$$2000 = CE_1^2 = C \times (10 \text{ V/m})^2$$
on the first strip and

$$N = CE_2^2 = C \times (30 \text{ V/m})^2$$
at the other strip. Dividing the second equation by the first, we have

$$\frac{N}{2000} = \frac{30^2}{10^2}$$
or $N = 18000$ photons.

39.14
We know that the squared modulus $|\psi(x)|^2$ must normalize to 1 (the particle must be somewhere). In other words, the area under the curve $f(x) = |\psi(x)|^2$ must equal 1. Using simple geometry, we may write

$$2a = 1 \quad \text{or} \quad a = \frac{1}{2}$$

Furthermore, the probability of the electron being between $x = 1.0$ nm and $x = 2.0$ nm is equal to the area under the curve $f(x) = |\psi(x)|^2$ on this interval. We easily have.

$$P(1.0 \text{ nm} \leq x_e \leq 2.0 \text{ nm}) = \frac{1}{2} \left( \frac{a}{2} \right) (1) = \frac{1}{2} \left( \frac{1}{2} \right) (1) = \frac{1}{8}$$

39.44
We know that the Heisenberg uncertainty principle gives us the constraint

$$\Delta x \Delta p \geq \frac{h}{2}$$

We know that the particle is in a box of length 1 mm. We therefore have $\Delta x = 1 \text{ mm} = 10^{12} \text{ fm}$ and $\Delta p = m\Delta v$. We therefore have

$$m\Delta v \geq \frac{h}{2\Delta x}$$

If we consider that the mass energy of sodium is about 23 times that of hydrogen ($m_{He}c^2 = 938$ MeV), then this gives us a minimum range of velocities $\Delta v$ as

$$\Delta v \geq \frac{h}{2m\Delta x} = \frac{h}{2m\Delta x} \frac{c^2}{c^2} = \frac{hc}{2|m|c^2} \Delta x \frac{c}{c} = \frac{(1240 \text{ MeV} \cdot \text{fm})}{2[23(938 \text{ MeV})](10^{12} \text{ fm})} (3 \times 10^8 \text{ m/s})$$

$$= 8.6 \times 10^{-6} \text{ m/s}$$

If we now assume that the root mean square velocity of these sodium particles is half of what we found for the range of velocities, then we have

$$v_{rms} = 4.3 \times 10^{-6} \text{ m/s}$$
We can then use equation 18.26

\[ v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \]

or

\[
T = \frac{m}{3k_B} v_{\text{rms}}^2 = \frac{m}{3k_B} v_{\text{rms}}^2 \frac{c^2}{n^2} = \frac{mc^2}{3k_B} \left( \frac{v_{\text{rms}}}{c} \right)^2 = \frac{23 \text{ (938 MeV)}}{3 (8.61 \times 10^{-5} \text{ eV K}^{-1})} \left( \frac{v_{\text{rms}}}{c} \right)^2
\]

\[
= \frac{23 \text{ (938 MeV)}}{3 (8.61 \times 10^{-11} \text{ MeV K}^{-1})} \left( \frac{4.3 \times 10^{-6}}{3 \times 10^8} \right)^2 = 1.71 \times 10^{-14} \text{ K}
\]

40.2

Firstly, the wavelength of the released photons (\( \lambda = 1484 \text{ nm} \)) is in the infrared spectrum. They have an energy equal to energy of the \( 3 \rightarrow 2 \) transition for the particles in the box.

\[
E_{3\rightarrow2} = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{1484 \text{ nm}} = 0.835 \text{ eV}
\]

We know that the energy of the \( 3 \rightarrow 2 \) transition for electrons in the box is also given by

\[
E_{3\rightarrow2} = \frac{\hbar^2}{8mL^2} (3^2 - 2^2) = \frac{\hbar^2}{8mL^2} \frac{c^2}{2} (3^2 - 2^2) = \frac{(hc)^2}{8(mc^2)L^2} \quad (5) = \frac{5(1240 \text{ eV} \cdot \text{nm})^2}{8(511 \times 10^3 \text{ eV})L^2}
\]

By equating these two expressions, we have

\[
0.835 \text{ eV} = \frac{5(1240 \text{ eV} \cdot \text{nm})^2}{8(511 \times 10^3 \text{ eV})L^2}
\]

or

\[
L^2 = \frac{5(1240 \text{ eV} \cdot \text{nm})^2}{8(511 \times 10^3 \text{ eV})(0.835 \text{ eV})} = 2.25 \text{ nm}^2
\]

Taking a square root of both sides yields \( L = 1.5 \text{ nm} \).

40.26

We shall now model an atom as an electron in a box of length \( L = 0.10 \text{ nm} \). The energy levels are thus given by

\[
E_n = n^2 \frac{\hbar^2}{8mL^2} = n^2 \frac{\hbar^2}{8mL^2} \frac{c^2}{2} = n^2 \frac{(hc)^2}{8(mc^2)L^2} = n^2 \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8(511 \times 10^3 \text{ eV})(0.10 \text{ nm})^2} = n^2 (37.61 \text{ eV})
\]

As such, \( E_1 = 37.61 \text{ eV} \), \( E_2 = 150.45 \text{ eV} \), \( E_3 = 338.51 \text{ eV} \), and \( E_4 = 601.8 \text{ eV} \). We shall easily calculate wavelengths of the emission spectrum from \( \Delta E_{i\rightarrow j} = E_i - \frac{hc}{\lambda}. \) Thus, we have

\[
\lambda_{2\rightarrow1} = \frac{hc}{\Delta E_{2\rightarrow1}} = \frac{1240 \text{ eV} \cdot \text{nm}}{(150.45 \text{ eV} - 37.61 \text{ eV})} = 10.98 \text{ nm}
\]

\[
\lambda_{3\rightarrow1} = \frac{hc}{\Delta E_{3\rightarrow1}} = \frac{1240 \text{ eV} \cdot \text{nm}}{(338.51 \text{ eV} - 37.61 \text{ eV})} = 4.12 \text{ nm}
\]

\[
\lambda_{3\rightarrow2} = \frac{hc}{\Delta E_{3\rightarrow2}} = \frac{1240 \text{ eV} \cdot \text{nm}}{(338.51 \text{ eV} - 150.45 \text{ eV})} = 6.59 \text{ nm}
\]

\[
\lambda_{4\rightarrow1} = \frac{hc}{\Delta E_{4\rightarrow1}} = \frac{1240 \text{ eV} \cdot \text{nm}}{(601.8 \text{ eV} - 37.61 \text{ eV})} = 2.19 \text{ nm}
\]

\[
\lambda_{4\rightarrow2} = \frac{hc}{\Delta E_{4\rightarrow2}} = \frac{1240 \text{ eV} \cdot \text{nm}}{(601.8 \text{ eV} - 150.45 \text{ eV})} = 2.74 \text{ nm}
\]

\[
\lambda_{4\rightarrow3} = \frac{hc}{\Delta E_{4\rightarrow3}} = \frac{1240 \text{ eV} \cdot \text{nm}}{(601.8 \text{ eV} - 338.51 \text{ eV})} = 4.70 \text{ nm}
\]
All of these wavelengths are in the ultraviolet range. The fact that the Bohr energies depends on the choice of \( E = 0 \). This choice is immaterial. Physics depends on energy differences; not the absolute energies themselves.

40.38

Here we want the probability for an electron of energy \( E = 0 \) to tunnel through an energy barrier (the work function) of \( U_0 = 4.3 \text{ eV} \) that is 50 nm wide. This probability is given by equation 40.53.

\[
P = e^{-2w/\eta}
\]

where \( w = 50 \text{ nm} \) and \( \eta \) (the penetration depth) has the form from equation 40.41

\[
\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}} = \frac{\hbar}{2\pi\sqrt{2mcU_0}} = \frac{hc}{2\pi\sqrt{2(511 \times 10^3 \text{ eV})(4.3 \text{ eV})}} = 0.094 \text{ nm}
\]

We thus have

\[
P = e^{-2w/\eta} = e^{-2(50)/(0.084)} = 9.61 \times 10^{-518} \approx 0
\]

41.54

Now we have a \( P = 100 \text{ MW} \) laser releasing a \( \Delta t = 40.0 \text{ ns} \) long pulse with a wavelength of \( \lambda = 690 \text{ nm} \). We wish to know how many stimulated emissions were necessary to create this pulse. The total energy of the pulse is

\[
E = P\Delta t = (100 \times 10^6 \text{ J/s})(40 \times 10^{-9} \text{ s}) = 4.0 \text{ J} = 2.49 \times 10^{19} \text{ eV}
\]

We also know that each photon created by stimulated emission has energy

\[
E_{\gamma} = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{690 \text{ nm}} = 1.79 \text{ eV}
\]

The total number of photons (and therefore the total number of stimulated emissions) is

\[
N = \frac{E}{E_{\gamma}} = \frac{2.49 \times 10^{19} \text{ eV}}{1.79 \text{ eV}} = 1.38 \times 10^{19}
\]

40.22

Here we are given that a spherical droplet of 2.0 \( \mu \text{m} \) diameter (radius \( r = 1.0 \mu \text{m} \)) is moving with a speed of 1.0 \( \mu \text{m/s} \) in a box of length \( L = 20 \mu \text{m} \). The mass of the water droplet is then

\[
m = \rho V = \rho \frac{4}{3}\pi r^3 = \left(1000 \text{ kg/m}^3\right) \frac{4}{3}\pi (1 \times 10^{-6} \text{ m})^3 = 4.18 \times 10^{-15} \text{ kg}
\]

The kinetic energy can be equated to the energy state of a particle in a box

\[
\frac{1}{2}mv^2 = E = n^2 \frac{\hbar^2}{8mL^2}
\]

or

\[
n = \sqrt{\frac{4m^2L^2v^2}{\hbar^2}} = 2mvL/\hbar = 2 \left(4.18 \times 10^{-15} \text{ kg}\right) \left(10^{-6} \text{ m/s}\right) \frac{20 \times 10^{-6} \text{ m}}{6.63 \times 10^{-34} \text{ J s}} = 2.52 \times 10^8
\]

This quantum number is so high we can very safely use classical mechanics approximate the evolution of this system.
We are given the energies of two adjacent energy levels for a particle in a box of length 10 fm.

\[
E_n = n^2 \frac{\hbar^2}{8mL^2} = 32.9 \text{ MeV}
\]

\[
E_{n+1} = (n+1)^2 \frac{\hbar^2}{8mL^2} = 51.4 \text{ MeV}
\]

By dividing the second equation by the first, we easily have

\[
\frac{(n+1)^2}{n^2} = \frac{51.4}{32.9}
\]

Solving for \(n\), we get \(n = 4\) and \(n+1 = 5\). The wavelength of the photon emitted in the \(n+1 \rightarrow n\) transition has energy \(\Delta E = E_\gamma = \frac{hc}{\lambda}\). This gives us

\[
\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{ nm}}{(51.4 \times 10^6 \text{ eV} - 32.9 \times 10^6 \text{ eV})} = 6.70 \times 10^{-5} \text{ nm}
\]

To find the mass, let us begin with

\[
32.9 \text{ MeV} = E_4 = 4^2 \frac{\hbar^2}{8mL^2} = 2 \frac{\hbar^2}{mL^2} \frac{c^2}{e^2} = 2 \frac{(hc)^2}{(me^2)L^2} = 2 \frac{(1240 \text{ MeV fm})^2}{(me^2)(10 \text{ fm})^2}
\]

Solving for the mass energy of the particle \(E_0 = mc^2\), we have

\[
E_0 = mc^2 = 2 \frac{(1240 \text{ MeV fm})^2}{(32.9 \text{ MeV})(10 \text{ fm})^2} = 934.71 \text{ MeV}
\]

This is about the mass energy of a proton or neutron. No other particle is even close. In kilograms, the mass is around

\[
m_{p,n} \simeq 1.67 \times 10^{-27} \text{ kg}
\]