Electrodynamics Chapter 1 Andrew Robertson

32.30

Here we are given a proton moving in a magnetic field $\vec{B} = 0.5\hat{\imath}$ T at a speed of $v = 1.0 \times 10^7$ m/s in the directions given in the figures.

Part A

Here, the velocity is at an angle of 45 degrees to the x-axis toward the z-axis. The velocity is then given in component form as

$$\vec{v} = v\cos(45)\,\hat{\imath} + v\sin(45)\,\hat{k} = \left(\frac{1}{\sqrt{2}} \times 10^7\right)\hat{\imath} + \left(\frac{1}{\sqrt{2}} \times 10^7\right)\hat{k}$$

The magnetic force on this particle is computed via a cross product as follows:

$$\vec{F} = q\vec{v} \times \vec{B} = q\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \left(\frac{1}{\sqrt{2}} \times 10^7\right) & 0 & \left(\frac{1}{\sqrt{2}} \times 10^7\right) \\ 0.5 & 0 & 0 \end{vmatrix} = q\left(\frac{1}{2\sqrt{2}} \times 10^7\right)\hat{\jmath}$$

Remembering that the charge on a proton is 1.60×10^{-19} C, we get a force of

$$\vec{F} = q\left(\frac{1}{2\sqrt{2}} \times 10^7\right)\hat{\jmath}\ \text{ N} = \left(5.65 \times 10^{-13}\right)\hat{\jmath}\ \text{ N}$$

Part B

Here, the velocity is in the direction of the negative x-axis. The velocity is then given in component form as

$$\vec{v} = -v\hat{\imath} = -1.0 \times 10^7\hat{\imath}\ \text{ m/s}$$

The magnetic force on this particle is computed via a cross product as follows:

$$\vec{F} = q\vec{v} \times \vec{B} = q\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1.0 \times 10^7 & 0 & 0 \\ 0.5 & 0 & 0 \end{vmatrix} = \vec{0}$$

Exactly as expected; A magnetic field exerts no force on a charged particle moving in the direction (or exact opposite direction) of the field lines.

32.36

We now wish to know the magnetic field strength and direction that will levitate the 2.0g wire in the figure. This wire is in a 10cm long region of space wherein the magnetic field is applied. The magnetic force $\vec{F}_B$ must exactly cancel the weight $\vec{W}$ of the wire in order to levitate it. That means the force must be pointing upward while the current is going from right to left. Using the right-hand rule, we know that this means the $\vec{B}$ field must be pointing out of the page. We still need to know how strong the field must be to completely cancel out gravity's influence. We know that the strength of the magnetic force must match that of the weight.

$$\left|\vec{F}_B\right| = \left|\vec{W}\right| = mg = (0.002\text{ kg})\left(9.8\text{ m/s}^2\right) = 0.0196\text{ N}$$

We also know that the form of the the magnetic force on a wire carrying current perpendicular to the magnetic field.

$$\left|\vec{F}_B\right| = ILB = (1.5\text{ A})(0.1\text{ m})\,B = 0.15B$$

Setting this equal to mg and solving for B gives us

$$B = \frac{mg}{IL} = \frac{0.0196}{0.15} = 0.13\text{ T}$$

32.76

Here we are given a nonuniform magnetic field acting on a loop of current carrying wire (which lies perpendicular to the page). We can tell from the right hand rule that the magnetic force $d\vec{F}_B$ on any infinitesimal segment of the ring will point upward at an angle of $\alpha = 90 - \theta$ to the vertical axis. See the diagram at the end of the document.

The problem informs us that the net force will be upward on the vertical axis (because of the cylindrical symmetry of the problem). As such, we need only concern ourselves with the upward component of $d\vec{F}_B$. That component is given by

$$dF_{up} = \cos(90-\theta)\left|d\vec{F}_B\right| = \cos(90-\theta)\,IB\ dl$$

where dl is an infinitesimal length of the ring. We can now get the total force by integrating along dl from 0 to the circumference of the ring $2\pi R$.

$$\left|\vec{F}_{net}\right| = F_{up} = \int_0^{2\pi R}\cos(90-\theta)\,IB\ dl = \cos(90-\theta)\,IB\int_0^{2\pi R} dl = 2\pi R\cos(90-\theta)\,IB$$

Part B

If $R = 0.02$ m, $I = 0.5$ A, $B = 200\times10^{-3}$ T, and $\theta = 20°$, we get the following for the net force.

$$\left|\vec{F}_{net}\right| = 2\pi R\cos(90-\theta)\,IB = 2\pi(0.02)\cos(90-20)(0.5)\left(200\times10^{-3}\right) = 0.0042\text{ N}$$

32.80

Here we wish to find the magnetic field strength that will cancel the torque exerted by a 50g weight hanging on a current carrying loop 2.5 cm from the axis of rotation. If W is the weight of the block and $r = 2.5$ is the distance from the axis of rotation, we know that the torque exerted by the block has a magnitude of

$$\tau_b = Wr\sin(90) = mgr\sin(90) = (0.05)(9.8)(0.025)(1) = 0.0122\text{ Nm}$$

Similarly, the magnetic torque on a loop in a magnetic field is equal to the cross product of its magnetic moment μ and the field $\vec{B}$. Since the angle between
$\vec{B}$ and the $\vec{\mu} = NI\vec{A}$ (where $\vec{A}$ is the loop's normal area vector) is 90 degrees, we have

$$|\vec{\tau}_B| = \left|\vec{\mu}\times\vec{B}\right| = \mu B\sin(90) = NIAB\sin(90) = (10)(2.0)(0.05*0.1)B = 0.1B$$

Setting these two torque magnitude equals to each other, we have

$$B = \frac{mgr}{NIA} = \frac{0.0122}{0.1} = 0.122\text{ T}$$

32.41

Here we need only apply the formula for the magnitude of the magnetic torque on a loop of current carrying wire.

$$|\vec{\tau}_B| = \left|\vec{\mu}\times\vec{B}\right| = \mu B\sin(30) = IAB\sin(30) = \left(500*10^{-3}\text{ A}\right)\left(0.05^2\text{ m}^2\right)(1.2\text{ T})\sin(30) = 7.5*10^{-4}\text{ Nm}$$

32.73

We are given a long current carrying wire suspended by threads and deflected from the vertical by $10°$ because of the presence of a magnetic field. We wish to find the strength and direction of that field. The right hand rule immediatel gives us that the direction must be straight downward. Now to compute it's strength, we have the following free body diagram. We shall look down the length of the wire from one end. Notice that if the strings are deflected by $10°$, then the tension must be applied to the wire at an angle of $90 - 10 = 80°$. From the diagram at the end of the document, we can see that the forces must cancel in the vertical direction.

$$T \sin(80) - mg = 0$$

and in the horizontal direction.

$$T \cos(80) - F_B = 0$$

Rearranging, we have

$$\begin{aligned} mg &= T \sin(80) \\ F_B &= T \cos(80) \end{aligned}$$

Dividing the top equation by the bottom, we have

$$\frac{mg}{F_B} = \tan(80)$$

or

$$F_B = \frac{mg}{\tan(80)}$$

To get the magnetic field, we shall make the substitutions

$$F_B = ILB \text{ and } mg = \lambda L g$$

where $\lambda = 0.05$ kg/m is the linear mass density and $F_B = ILB$ because $\vec{B}$ is perpendicular to the wire. Subsituting these into the equation above, we have

$$ILB = \frac{\lambda L g}{\tan(80)}$$

Solving for B, we have

$$B = \frac{\lambda g}{I \tan(80)} = \frac{(0.05)(9.8)}{10 \tan(80)} = 0.0086 \text{ T}$$

32.66

We are now given an electron entering a region with a uniform 30 mT magnetic field in the positive z direction. The electron has a speed $v = 5.0 * 10^6$ m/s directed at an angle $30°$ above the x-y plane. We want the radius r and the pitch p of the spiral trajectory the electron will follow. We know that the radius of cyclotron orbit is given by

$$r_{cyc} = \frac{mv_\perp}{qB}$$

where $v_\perp$ is the magnitude of the velocity component that is perpendicular to the magnetic field. In our case, this component is equal to the component of $\vec{v}$ in the xy plane.

$$v_\perp = \sqrt{v_x^2 + v_y^2} = \sqrt{v^2 - v_z^2} = \sqrt{v^2 - v^2 \sin^2(30)} = v\sqrt{1 - \sin^2(30)} = v\frac{\sqrt{3}}{2} = 4.33 * 10^6 \text{ m/s}$$

For the radius, this gives us

$$r_{cyc} = \frac{m_e v_\perp}{eB} = \frac{\left(9.11 * 10^{-31} \text{ kg}\right)\left(4.33 * 10^6 \text{ m/s}\right)}{(1.6 * 10^{-19} \text{ C})(30 * 10^{-3} \text{ T})} = 8.21 * 10^{-4} \text{ m}$$

Next, we realize that the pitch p is equal to how far upward in the z direction the electron goes in the time it takes to complete one cyclotron orbit. That time, called the period T, is the reciprocal of the cyclotron frequency f given in equation 32.20.

$$p = v_z T = v\sin(30)\frac{1}{f} = v\sin(30)\frac{2\pi m_e}{qB} = 0.0029 \text{ m}$$

32.40

Simply applying the right-hand rule to both of these situations indicates the magnetic force on any straight side of the loop is opposed by the magnetic force on the opposite side. Because that force is equal to $F_B = ILB$, which is the same for each side, we can be sure that the net force on both of these loops is zero. Similarly, the torque on each loop is

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

However, $\vec{\mu}$ and $\vec{B}$ are either parallel or antiparallel to each other for both loops, hence their cross products vanish. Thus, there is no torque on these loops either.

part B

For loop 1, we see that $\vec{\mu}$ is parallel to $\vec{B}$. However, $\vec{\mu}$ is antiparallel to $\vec{B}$ for loop 2. So when loop 2 is perturbed slightly from equlibrium by rotating its moment $\vec{\mu}$ slightly toward the vector $\vec{B}$, the torque exerted will drag $\vec{\mu}$ further toward $\vec{B}$ and further away from equilibrium much like gravity pulls a rolling ball further from its equilibrium point at the top of a hill.

Similarly for loop 1, if perturbed slightly by rotating its moment $\vec{u}$ slightly away from $\vec{B}$, the magnetic torque exerted will try to pull $\vec{u}$ back toward $\vec{B}$ and toward equilibrium. This is like how gravity pulls a rolling ball back into a well or basin when it tries to escape.

32.70

Here we are given a mass spectrometer. The accelerating potential ΔV imparts a kinetic energy equal to $q\Delta V$ to a charge particle before it enters the magnetic field.

$$KE = \frac{1}{2}mv^2 = q\Delta V$$

Solving for v, we have

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

It will then undergo a cyclotron orbit of radius

$$\frac{d}{2} = r_{cyc} = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}}$$

Solving for ΔV gives us

$$\Delta V = \frac{m}{2q}\left(\frac{qBr_{cyc}}{m}\right)^2 = \frac{m}{2q}\left(\frac{qBd}{2m}\right)^2 = \frac{(Bd)^2}{8}\left(\frac{q}{m}\right)$$

Thus, we may easily calcultate the potential difference ΔV necessary for detecting N_2^+

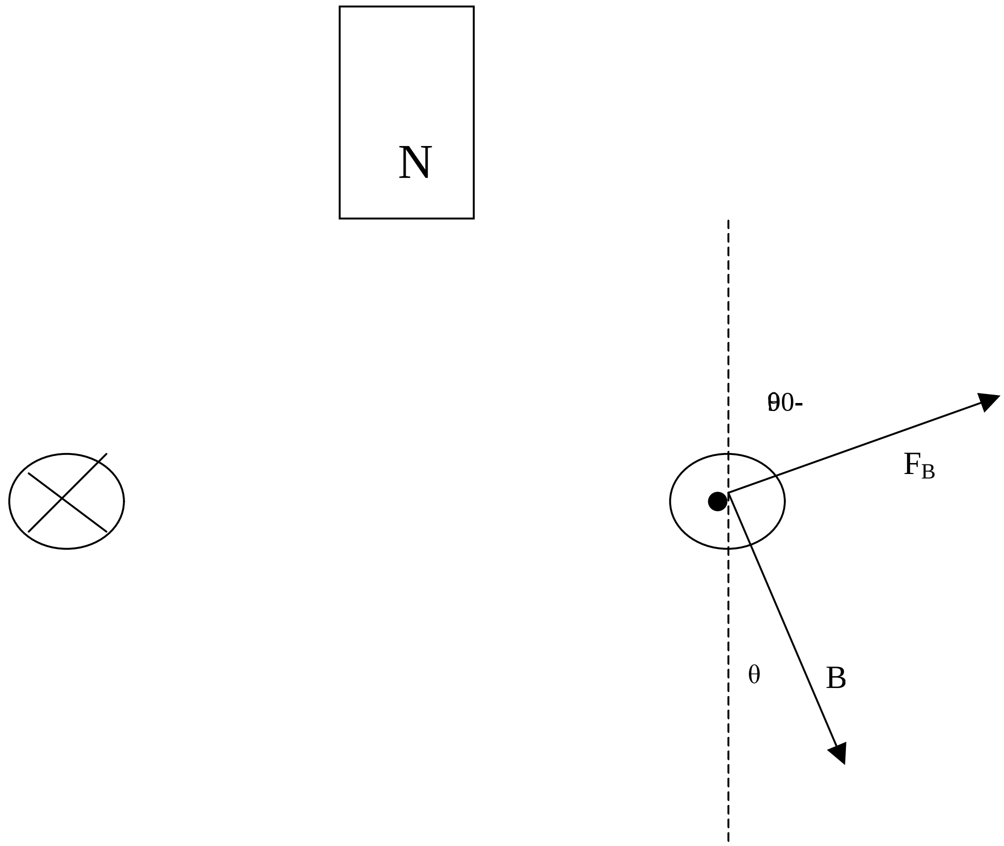
$$\Delta V = \frac{(0.2*0.08)^2}{8}\left(\frac{q}{m}\right) = \frac{(0.2*0.08)^2}{8}\left(\frac{15\left(1.60*10^{-19}\text{ C}\right)}{28.0062\left(1.66*10^{-27}\text{ kg}\right)}\right) = 1651.96 \text{ V}$$

for O_2^+

$$\Delta V = \frac{(0.2 * 0.08)^2}{8}\left(\frac{q}{m}\right) = \frac{(0.2 * 0.08)^2}{8}\left(\frac{17\left(1.60 * 10^{-19}\ \mathrm{C}\right)}{31.9898\,(1.66 * 10^{-27}\ \mathrm{kg})}\right) = 1639.08\ \mathrm{V}$$

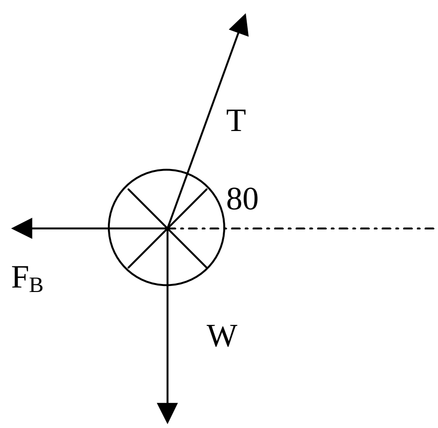
and lastly for CO^+

$$\Delta V = \frac{(0.2 * 0.08)^2}{8}\left(\frac{q}{m}\right) = \frac{(0.2 * 0.08)^2}{8}\left(\frac{15\left(1.60 * 10^{-19}\ \mathrm{C}\right)}{27.9949\,(1.66 * 10^{-27}\ \mathrm{kg})}\right) = 1652.62\ \mathrm{V}$$



Problem 32.76

Problem 32.73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