A probabilistic approach for safety risk analysis in metro construction

Limao Zhang \textsuperscript{a,b}, Miroslaw J. Skibniewski \textsuperscript{b,c}, Xianguo Wu \textsuperscript{a,*}, Yueqing Chen \textsuperscript{a}, Qianli Deng \textsuperscript{b}

\textsuperscript{a} School of Civil Engineering & Mechanics, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China
\textsuperscript{b} Department of Civil & Environmental Engineering, University of Maryland, College Park, MD 20742-3021, USA
\textsuperscript{c} Institute of Theoretical and Applied Informatics, Polish Academy of Sciences, Poland

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\textbf{A B S T R A C T}

This paper presents a probabilistic decision approach for safety risk analysis for metro construction in complex project environments. An expert confidence index is proposed for the fuzzy probability estimation of basic events, aiming to ensure the reliability of collected data during the expert investigation. An approach for defuzzification is developed in a precise way based on the representation theorem, attempting to overcome the limitations related to fuzzy linear approximations. A sensitivity analysis technique is utilized to evaluate the percentage contribution of the basic events to the failure of a top-event. A possibility based importance index, fuzzy importance measure, is deployed for the sensitivity analysis of basic event to reveal the critical basic events for reducing the risk limit. A typical hazard in tunnel cross passage construction (TCPC) on the example of Wuhan Yangtze Metro Tunnel construction is presented in a case study. The results demonstrate the feasibility of the proposed method and its application potential. The proposed method can be used as a decision support tool to provide guidelines for safety management in metro construction, and thus increase the likelihood of a successful project in a complex environment.

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1. Introduction

In the past ten years, metro construction has presented a powerful momentum for rapid economic development worldwide. Owing to various risk factors in complex project environments, safety violations occur frequently in metro construction worldwide (AFP, 2008; Schexnayder, 2007; Thomas, 2010; Yu, 2012). Obviously, metro construction entails a highly complicated project with large potential risks, which can bring enormous dangers to public safety.

Many accidents have led to the growing public concern for a priori Probabilistic Risk Assessment (PRA) in relation to the metro construction safety (Abdelgawad and Fayek, 2010). PRA plays an important role in safety management process, aiming to illustrate the risk factors’ contribution to the occurrence of an accident. Critical potential risks and risk factors can then be identified to help project engineers determine the main safety checkpoints in the construction phase. There are several methodologies proposed for risk analysis, such as hazard and operability study (HAZOP), functional hazard analysis, failure modes and effect analysis (Pillay and Wang, 2003). Most of the traditional methodologies are based on qualitative analysis tools, and are normally used for preparing feedback for a fault tree analysis (FTA) (Yuhua and Datoa, 2005).

FTA is one of the most effective techniques for estimating the frequency of occurrence of hazardous events in PRA (Deshpande, 2011; Labib and Read, 2013). According to “Guidelines for Tunneling Risk Management” published by the International Tunneling Association (ITA) in 2004, FTA was highly recommended as a tool for risk analysis and assessment (Degn Eskesen et al., 2004). In conventional FTA, the occurrence probability of basic events was always regarded as a crisp value. However, in engineering practice, it is difficult or nearly impossible to obtain exact values of probability due to the lack of sufficient data (Mentes and Helvacioglu, 2011). Singer (1990) indicated that conventional FTA resulted in insufficient information concerning the relative frequencies of hazardous events in many cases. Therefore, a group decision making method is generally employed to assess the occurrence probability of basic events. The uncertainties can then be considered in term of intervals or fuzzy numbers (Horcik, 2008). Fuzzy arithmetic provides a successful tool to solve engineering problems with uncertain parameters (Hanss, 1999). Nowadays, fuzzy numbers are widely used in engineering applications because of their suitability for representing uncertain information (Boukezzoula et al., 2007). Thus, fuzzy fault tree analysis has experienced rapid development recently.

Metro construction can be regarded as a typical large-scale system, associated with enormous potential risks to public safety (Shi et al., 2012). Any kind of information loss or calculation errors are therefore unacceptable for decision support analysis, and the requirement for the precise estimation of failure probabilities is
in great need in metro engineering practice. In current fuzzy based probability analysis, all collected data is entered for decision analysis without any data reliability evaluation, resulting in inaccurate problems in subsequent computation (Ferdous et al., 2009). Meanwhile, the membership functions of the multiplied fuzzy numbers have to be approximated by linear functions at the defuzzification stage (Mentes and Helvacioglu, 2011). Such fuzzy linear approximations are likely to generate considerable information loss and certain deviations, leading to errors in risk analysis and decision making (Hanss, 1999). To overcome those limitations, a fuzzy decision analysis approach for safety management in metro construction in complex environments is proposed in this paper. The expert confidence index is first introduced to ensure the reliability of collected data during the expert investigation, with the expert judgment ability and subjectivity taken into account. Then, an approach for defuzzification is developed based on the representation theorem. A possibility based importance index, called fuzzy importance measure, is deployed for the sensitivity analysis of basic events, attempting to reveal the critical basic events so as to reduce the risk limit. Finally, a typical hazard in tunnel cross passage construction (TCPC) such as one located in the Wuhan Yangtze Metro Tunnel is used for a case study. The results demonstrate the feasibility of the proposed method, as well as its application potential.

This paper is organized as follows. The fundamental theory and the proposed analysis method are introduced in Section 2. Section 3 highlights two steps in the proposed method, namely the fuzzification and defuzzification processes. Improvements are presented in response to the limitations of the current approaches. In Section 4, the proposed method is applied to fuzzy decision support in safety management. The conclusions are presented in Section 5.

2. Fuzzy decision analysis methodology

2.1. Fault tree analysis

A fault tree (FT) is a structured logical diagram composed of root causes (basic events) and a top event. The concept of fault tree analysis (FTA) was first introduced in 1961 by H.A. Watson of Bell Laboratories. Since then, significant contributions have been made by developing algorithms to solve fault trees (Ericson and Ll, 1999). FTA employs two basic assumptions. The first is related to likelihood values of input events, and the second is concerned with interdependence among basic events (Ferdous et al., 2010). Logical signs, such as “OR” and “AND” gates, are used to represent relationships among various events (see Figs. 1 and 2). FTA is a deductive oriented systematic and graphical analysis technique used to determine the occurrence probability of an event. FTA is currently used within various industries for conducting qualitative and quantitative assessment of risk events (Zeng and Skibniewski, 2012). The probability of the top event (TE) is calculated by propagating probabilities among events until TE is reached. To do so, enough historical data is required to estimate the probability of basic events. However, having sufficient data to derive probability distributions is often difficult in the construction industry (Abdelgawad and Fayek, 2010).

2.2. Fuzzy set theory and fuzzy fault tree analysis

Fuzzy set theory (FST) was first introduced by Zadeh (1965) in order to deal with uncertainty due to imprecision and vagueness. FST provides a basis to generate powerful problem-solving techniques with wide applicability especially in the field of decision making (Chen and Chen, 2007). A fuzzy set is usually indicated by a tilde “~” where X is characterized by a membership function \( u_p(x) \) with an interval [0, 1]. The function \( u_p(x) \) represents the membership value of x in P. In general, FST uses triangular, trapezoidal and Gaussian fuzzy numbers, which convert the uncertain numbers into fuzzy numbers (Abbasbandy and Hajjari, 2010). In order to analyze a safety problem, triangular fuzzy numbers are often utilized to provide more precise descriptions and to obtain more accurate solutions. Thus, in this paper, triangular fuzzy numbers are applied for representing the probabilities of basic events. A fuzzy set \( P = (m - a, m, m + b) \) is called a triangular fuzzy number if its membership function is given by Eq. (1).

\[
u_p(x) = \begin{cases} 0, & x \leq m - a \\ \frac{x - m + a}{b}, & m - a < x \leq m \\ 1, & x = m \\ \frac{m + b - x}{b}, & m \leq x \leq m + b \\ 0, & x \geq m + b \end{cases}
\]

where a, b, and c are the real numbers. A triangular fuzzy number \( P \) is shown in Fig. 3. The constants \( [m - a, m + b] \) give the lower and upper bounds of the available area, reflecting the fuzziness of the evaluation data.

During the fault tree construction, analysts are confronted with insufficient data concerning probabilities of basic events. In construction practice, the occurrence of an extremely hazardous event is rare, and therefore, the data would be rare. In the absence of sufficient data, it is necessary to work with rough estimates of

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**Fig. 1.** “OR” gate representation in FTA.

**Fig. 2.** “AND” gate representation in FTA.

**Fig. 3.** Membership function of a triangular fuzzy number \( P \).
probabilities (Liu and Tsai, 2012). Obviously, under such uncertain circumstances, it is considered inappropriate to use conventional FTA for computing the system failure probability. FST offers the frame of analysis that could deal with imprecision in input failure probabilities for the estimation of top event probability, and such analysis is termed fuzzy fault tree analysis (FFTA) (Mentes and Helvaciglu, 2011). The fuzzy probability of the TE is represented by $P_{TE}$, $P_{TE}$ of the n inputs connected by an OR gate (see Fig. 1) can be defined by Eq. (2), $P_{TE}$ of the n inputs connected by an AND gate (see Fig. 2) can be defined by Eq. (3).

$$P_{TE}^0 = 1 - \left[ (1 - P_{x_1}) \otimes (1 - P_{x_2}) \otimes \cdots \otimes (1 - P_{x_n}) \right]$$  \hspace{1cm} (2)

$$P_{TE}^{AND} = P_{x_1} \otimes P_{x_2} \otimes \cdots \otimes P_{x_n}$$  \hspace{1cm} (3)

### 2.3. Fuzzy sensitivity analysis

In previous FFTA studies, the TE probability merely gives an idea about system conditions, regardless of the percentage contribution of each basic event. Determining the importance level of different basic events is essential for decision analysis. Vesley et al. (2002) indicated that less than 20% of basic events are responsible for more than 90% of the probability of the TE. The sensitivity analysis is therefore applied to identify the weakest components of the system (Contini et al., 2000).

To determine the importance level of different root causes, Suresh et al. (1996) first introduced a fuzzy importance measure (FIM) by means of the Euclidean distance. In this paper, FIM is utilized to carry out the sensitivity analysis, attempting to reveal the critical basic events so as to reduce the risk limit. The basis of the FIM calculation is to first assess the TE fuzzy probability ($P_{TE}$), assuming that all basic events occur. Next, each basic event $x_i$ is eliminated (i.e., by setting $P_{x_i} = 0$ for the event $x_i$) and again calculating the TE fuzzy probability ($P_{TE}$). Subsequently, the FIM of the event $x_i$, denoted by $FIM_i$, can be calculated by Eq. (4). According to Abdelgawad and Fayek (2010), results showed that FIM gave more logical results than other indexes, based on the risk coordinator’s assessment.

$$FIM_i = \left[ \frac{P_{TE} - P_{TE}}{P_{TE}} \right] \times 100\%$$  \hspace{1cm} (4)

### 2.4. A step-by-step procedure for decision analysis

Quantitative FFTA is a time-consuming activity and requires several steps. For the implementation of the fuzzy decision analysis for safety management in complex project environments, a systematic methodology is developed, as seen in Fig. 4. In the proposed approach, the following steps are adopted:

- **Step 1. Risk/Hazard identification**: Carry out the preliminary risk mechanism analysis for the problem, reveal the potential risks/risk factors and their causal relationships, and then identify the top-event and sub-events.
- **Step 2. Fault tree construction**: Identify the potential failure-consequence scenario of the top event, develop a failure logic, and then build up a fault tree using the basic events.
- **Step 3. Probability assessment (Fuzzification)**: Carry out the expert investigation for the probability estimation of basic events, gather the data, transform the linguistic and fuzzy expressions into fuzzy numbers, and then calculate the fuzzy probability of basic events based on fuzzification process.
- **Step 4. Fuzzy based risk analysis (Defuzzification)**: Calculate fuzzy failure probabilities using the probabilities of basic events, and use the fuzzy importance measure for sensitivity analysis. Convert the resulting fuzzy probability into a crisp value based on the defuzzification process, and then rank the calculating results.
- **Step 5. Decision making**: Figure out the critical sensitivity factors based on risk analysis results, propose relevant control measures for risk response, and make decisions for monitoring and reviewing process.

Steps 1, 2, and 5 rely on the elicitation of knowledge from experts using a standard technique (e.g., Chapman (1998)). The next section highlights steps 3 and 4.

### 3. Fuzzification and defuzzification process

Fuzzification and defuzzification both play a crucial role in the fuzzy decision analysis. Fuzzification attempts to define the basic event data into a fuzzy probability set and uses them in subsequent computation, while defuzzification is to obtain a precise top event probability (Ferdous et al., 2009). Owing to the high potential risks for the metro construction in complex environments, the fuzzy probability analysis should meet the highly required precision for the purpose of safety management in the metro construction practice. However, the precision of the calculated results is significantly affected due to the limitations in current approaches for fuzzification and defuzzification.

#### 3.1. Fuzzification

#### 3.1.1. Limitations on current fuzzy probability assessment

In fuzzy based probability analysis, the imprecise failure probabilities of basic events are refined by characterizing the basic event with a suitable membership function. It is difficult to have an exact estimation of the failure rate due to the lack of sufficient data. Therefore, a group decision making method is generally employed to define the linguistic terms to assess the fuzzy probability of occurrence of basic events. There mainly exist two deficiencies during the current fuzzy probability assessment process as follows:

![Fig. 4. Fuzzy decision analysis procedure for safety management in tunnel construction.](image-url)
(1) In traditional expert investigation, all collected survey data is entered into the fault tree without any kind of data reliability evaluation. In fact, most interviewed individuals have different confidence levels for their subjective judgments according to their educational levels, working years and risk attitudes. Thus, a certain deviation exists in the data reliability among different interviewed individuals. For instance, the reliability of the investigation data obtained from a project engineer with 30 years of working experience is comparatively higher than that from an engineer with 5 years of working experience. It is therefore necessary to carry out the data reliability evaluation with expert ability and subjectivity fully considered.

(2) During the division of probability intervals, five linguistic terms [very low (VL), low (L), medium (M), high (H), and very high (VH)] are commonly used to assess the probability occurrence of an event. The current probability span is excessively large within a single interval. For instance, the scope of “H” ranges from 21.5% to 67% (see Abdelgawad and Fayek, 2010). Such rough intervals division cannot meet the required precision for failure probability assessment of basic events in metro engineering practice.

3.1.2. Expert confidence index

Expert confidence index is proposed to reveal the reliability of the data obtained from interviews with various individuals. For one thing, the expert judgment ability needs to be first taken into account. In construction practice, it is generally considered that the judgment ability of individuals tends to become increasingly sophisticated and stable with the accrual of educational background and working experience. In other words, the judgment ability level, denoted by $\zeta$, is improved accordingly. The judgment ability is divided into four levels, represented by “I, II, III, IV” as seen in Table 1. The level “I” with a score of 1.0 stands for the highest reliability for the expert judgment ability. For another thing, the expert confidence index involves a kind of subjective measure related to their judgments during the expert investigation. For the purpose of data analysis and normalization processing, the average occurrence probability of a specific basic event lying in the $k$th interval with a subjective reliability level $\psi$, the probability fuzzification process, as seen in Eqs. (8)–(10).

In an actual investigation, there are $S$ experts involved in the investigation. For the purpose of data analysis and normalization processing, the average occurrence probability of a specific basic event lying in the $i$th interval is calculated to be $P_i$ ($1 \leq i \leq 17$) using Eq. (7). According to the Gaussian distribution patterns of random variables (Montgomery et al., 2009), the occurrence probability tends to fluctuate around its expectation, and decrease gradually as it goes far away from the expectation. Thus, a simplified formula concerning the distribution of residual probability $1 - \theta$ among other intervals is presented in this paper, as seen in Eq. (6).

$$\theta = \zeta \times \psi$$

3.1.3. Probability interval division and expert investigation

To reach the highly required precision for the occurrence probability of the top event in the metro construction practice, the occurrence probability of basic events is divided into 17 intervals, represented by “1–17”. As seen in Table 2, the $k$th interval is defined by $[a_k, a_{k+1}]$ together with a mean $c_k$ ($1 \leq k \leq 17$). During the expert investigation, the purpose of questionnaires as seen in Table 3 is to collect the information related to occurrence probability interval $K$ and subjectivity reliability level $\psi$. For “Probability interval $K$”, interviewed individuals were required to fill in a number ranging from 1 to 17 (as seen in Table 2). For “Subjectivity reliability level $\psi$", they were required to fill in a number ranging from 1.0 to 0.6. If they fail to evaluate the data reliability within a reliability degree of more than 0.6 by themselves, that field can be left blank.

3.1.4. Data gathering

Assuming one considers the occurrence probability of a specific basic event is in the $k$th interval with a subjective reliability $\psi$, the expert confidence index $\theta$ for the event lying in the $k$th interval can then be calculated by Eq. (5). In general, $\theta$ is lesser than 1, which means that the event has a residual probability of $1 - \theta$ lying in other intervals. This kind of information is often lost, regardless of the potentially useful information. According to the Gaussian distribution patterns of random variables (Montgomery et al., 2009), the occurrence probability tends to fluctuate around its expectation, and decrease gradually as it goes far away from the expectation. Thus, a simplified formula concerning the distribution of residual probability $1 - \theta$ among other intervals is presented in this paper, as seen in Eq. (6).

$$p_i = \left\{ \begin{array}{ll}
\frac{1}{\theta} \left( \frac{1}{2} \right)^{i-1} & 1 \leq i < k - 1 \\
\frac{1}{\theta} \left( \frac{1}{2} \right)^{k-1} & k + 1 \leq i \leq 17 
\end{array} \right.$$

3.1.5. Probability fuzzification

In an actual investigation, there are $S$ experts involved in the investigation. For the purpose of data analysis and normalization processing, the average occurrence probability of a specific basic event lying in the $i$th interval is calculated to be $P_i$ ($1 \leq i \leq 17$) using Eq. (7). According to the Gaussian distribution patterns of random variables, the data reliability for the random variable lying in the interval $[E(P) - 3\sigma, E(P) + 3\sigma]$ reaches up to 99.7%, where $E(P)$ stands for the expectation and $\sigma$ stands for the standard deviation (Montgomery et al., 2009). The above mentioned principle is also called the “3σ rule”. In this paper, the “3σ rule” is adopted for the probability fuzzification process, as seen in Eqs. (8)–(10).

$$p_i = \sum_{i=1}^{S} p_i/S$$

$$m = E(P) = \sum_{i=1}^{17} (c_i \times P_i)$$

$$\sigma = \sqrt{D(P)} = \sqrt{\sum_{i=1}^{17} [(c_i - E(P))^2 \times p_i]}$$

<table>
<thead>
<tr>
<th>Level</th>
<th>Expert description</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1. Domain experts with more than 30 years of working experience</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2. Professors within the research field of metro construction</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>1. Domain experts with 10–20 years of working experience.</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>2. Associate professors within the research field of metro construction</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>1. Domain experts with 5–10 years of working experience</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>2. Assistant professors within the research field of metro construction</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>1. Domain experts with 1–5 years of working experience</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Table 2  
Divisions of occurrence probability intervals.  

<table>
<thead>
<tr>
<th>Interval ID (k)</th>
<th>Left boundary ((a_k)) (%)</th>
<th>Mean ((c_k)) (%)</th>
<th>Right boundary ((a_{k+1})) (%)</th>
<th>Interval ID (k)</th>
<th>Left boundary ((a_k)) (%)</th>
<th>Mean ((c_k)) (%)</th>
<th>Right boundary ((a_{k+1})) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.05</td>
<td>0.1</td>
<td>10</td>
<td>9.0</td>
<td>11.0</td>
<td>13.0</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
<td>11</td>
<td>13.0</td>
<td>15.0</td>
<td>17.0</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0.35</td>
<td>0.5</td>
<td>12</td>
<td>17.0</td>
<td>21.0</td>
<td>25.0</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.75</td>
<td>1.0</td>
<td>13</td>
<td>25.0</td>
<td>30.0</td>
<td>35.0</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>14</td>
<td>35.0</td>
<td>40.0</td>
<td>45.0</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>15</td>
<td>45.0</td>
<td>52.5</td>
<td>60.0</td>
</tr>
<tr>
<td>7</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
<td>16</td>
<td>60.0</td>
<td>70.0</td>
<td>80.0</td>
</tr>
<tr>
<td>8</td>
<td>5.0</td>
<td>6.0</td>
<td>7.0</td>
<td>17</td>
<td>80.0</td>
<td>90.0</td>
<td>100.0</td>
</tr>
<tr>
<td>9</td>
<td>7.0</td>
<td>8.0</td>
<td>9.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the above two defuzzification approaches, both consider that the calculated result remains a triangular fuzzy number. However, that is not the case, and the main problem lies in the fuzzy operation. Specifically, assuming two triangular fuzzy numbers \(\tilde{A}_1 = (m_1 - a_1, m_1, m_1 + b_1)\) and \(\tilde{A}_2 = (m_2 - a_2, m_2, m_2 + b_2)\), the following operators can be defined by Eqs. (11)-(13). As seen in Eqs. (11) and (12), either the addition or subtraction result remains an exact triangular fuzzy number. However, the multiplication result (see Eq. (13)) is no longer an exact triangular fuzzy number (Gao et al., 2009). In addition, numerous scholars considered the multiplication result as an approximation triangular fuzzy number for simplified computation consideration. However, this kind of fuzzy linear approximation operation is likely to generate certain deviations and information loss, leading to errors in risk analysis and decision making, especially when high precision is required for safety management in metro engineering practice.

\[
a = b = 3\sigma
\]  

where \(c_i\) refers to the mean of \(i\)th probability interval as shown in Table 2; \(a\), \(m\), and \(b\) refer to characteristic values of a triangular fuzzy number as shown in Fig. 3.

3.2. Defuzzification

3.2.1. Limitations on current defuzzification approaches

In the aforementioned fuzzy based risk analysis, the calculated occurrence probability for the TE is also a fuzzy number, represented by \((m - a, m, m + b)\). For the purpose of risk sensitivity analysis and evaluation, it is therefore necessary to provide a crisp value of the fuzzy probability at the defuzzification stage. Specifically, the objective of defuzzification is to determine an exact value as the representative of the fuzzy number, denoted by “half”. Current defuzzification method consists of two approaches, namely a median based approach and a centroid based approach. The median based approach aims to obtain the abscissa value of the point through which the vertical line divides the area into two equal areas, as seen in Fig. 5(a). The centroid based approach aims to obtain the abscissa value of the gravity center of the area enclosed by fuzzy member function, as seen in Fig. 5(b). Obviously, when the membership function of the fuzzy number is nonlinear, the defuzzification tends to be inaccurate by means of coordinate difference. Therefore, the median based approach is presented in the case study later in this paper.

\[
\frac{1}{3} A_1 + \frac{2}{3} A_2 = (m_1 + m_2 - a_1 - a_2, m_1 + m_2, m_1 + m_2 + b_1 + b_2)
\]  

(11)

\[
\frac{1}{3} A_1 - \frac{2}{3} A_2 = (m_1 - m_2 - a_1 + a_2, m_1 - m_2, m_1 - m_2 + b_1 - b_2)
\]  

(12)

\[
\frac{1}{3} A_1 \otimes \frac{2}{3} A_2 \approx ((m_1 - a_1) \times (m_2 - a_2), m_1 \times m_2, (m_1 + b_1) \times (m_2 + b_2))
\]  

(13)

3.2.2. Representation theorem

To overcome the limitations related to the fuzzy linear approximation operation mentioned above, representation theorem, one of the three fundamental theorems in the fuzzy set theory, is adopted. Representation theorem provides a powerful tool for dealing with the uncertain relation between the fuzzy set and cantor set, and it is widely used in fuzzy set construction (Wu, 2012), interval valued structure (Yuan et al., 2011) and other fields. The description of the representation theorem is as follows:

![Fig. 5. Current approaches for defuzzification.](image)
Representation theorem: If $H : (0, 1) \rightarrow E$, $\lambda \mapsto H(\lambda) = [m_1, n_1] \neq \emptyset$, $\lambda_1 < \lambda_2 = \{m_1, n_1\} \geq \{m_2, n_2\}$, $A$ is a fuzzy number with an interval $[m_1, n_1]$, $[m_2, n_2]$ stands for the $\lambda$ cut set interval of $A$, then:

1. $\bar{A} = \bigcup_{\lambda = 0}^{1} H(\lambda)$;
2. $\bar{A}_\lambda = \bigcap_{\lambda = 0}^{1} H(\lambda)$, $\lambda > 0$, $\lambda = \left(1 - \frac{1}{n}\right)$;
3. $\bar{A} = (\{m_1, n_1\}, \{l_1, r_1\})$, and $m_\lambda = \lim_{n \to \infty} m_\lambda$, $n_\lambda = \lim_{n \to \infty} n_\lambda$, $l_\lambda(x) = \vee_{0 < \lambda < 1} \{\lambda | m_1 \leq x\}$, $r_\lambda(x) = \vee_{0 < \lambda < 1} \{\lambda | n_1 \geq x\}$.

As seen in Eq. (13), the multiplication result, denoted by $Q = A_1 \otimes A_2$, remains a fuzzy number with an interval of $[(m_1 - a_1) \times (m_2 - a_2), (m_1 + b_1) \times (m_2 + b_2)]$. Based on the representation theorem, the membership function of $Q$ is shown in Fig. 6. The membership degree $u_\lambda(x)$ is given to be 0 when $x$ lies at the abscissa of $(m_1 - a_1) \times (m_2 - a_2)$ or $(m_1 + b_1) \times (m_2 + b_2)$, while the membership value is given to be 1 when $x$ lies at the abscissa of $m_1 \times m_2$. Furthermore, the membership value $u_\lambda(x)$ is given to be $\lambda$ when $x$ lies at the abscissa of $m_1$ or $n_1$. Both $m_1$ and $n_1$ can be calculated by Eq. (14). After that, the nonlinear membership function between fuzzy numbers can be obtained, and then the Calculus method can be employed to calculate the area of the region enclosed by fuzzy member function (Stewart, 2010). Similarly, the membership function of the multiplication result of $n$ fuzzy triangular numbers $(n \geq 2)$ can also be achieved in this way. In the meantime, the precision of the function curve can be significantly improved by means of the fine division of $\lambda$. Generally, the result is considered as precise as to meet the inquired precision in safety management in metro construction when the precision of $\lambda$ reaches up to 0.001.

$$\begin{align*}
m_1 &= \prod_{j=1}^{2} (m_j - a_j(1 - \lambda)) \\
n_1 &= \prod_{j=1}^{2} (m_j + b_j(1 - \lambda))
\end{align*}$$

3.2.3. Comparison

As mentioned above, this kind of fuzzy linear approximation operation is likely to generate certain deviations. To determine how large the impact is on the defuzzification result, a simplified example is carried out for comparison. Assuming $A_1 = A_2 = (0.5, 1)$ and TE refers to the output of $A_1$ and $A_2$ connected by an OR gate, the fuzzy probability of $P_E$ is then calculated using Eqs. (2) and (3). Finally, the defuzzification results are shown in Fig. 7, including the approximate, precise and real value. Herein, the approximate value is calculated using the traditional fuzzy linear approximation operation, the precise value is calculated by the exact approach based on the representation theorem, and the real value is calculated using direct analytical method (see Gao et al. (2009)).

As seen in Fig. 7, the approximate value of 61.24% is 7.04% lower than the real value of 65.88%, and the precise value of 65.25% is 0.96% lower than the real value. For safety management in metro construction, an error exceeding 5% is unacceptable and it is likely to result in errors in decision making. Obviously, the proposed defuzzification approach is superior to the traditional fuzzy linear approximation approach. However, looking forward to calculating the real value by the analytical method is unrealistic because of the complexity in actual construction project conditions. Specifically, there are various basic events contributing to the high risk level of metro construction, and the multiplication of $n$ fuzzy numbers is involved in the computation process. The solution to an univariate cubic equation is challenging to date, not to mention the solution to a $n$th degree equation in one variable associated with complex logical structures. By contrast, the exact approach has the features of simple computation and better adaptability, and is therefore adopted in this research.

4. Fuzzy safety management in tunnel construction – a case study

With the advantage of high level of mechanization, fast progression of tunneling works and lesser environmental impact, shield tunneling technology is widely used in numerous domains, especially for metro construction (Li and Chen, 2012). Due to the complexity of construction technologies and the surrounding environment, numerous potential safety risks exist during the tunnel construction process. According to Ding et al. (2012), safety risks can be divided into three categories in tunnel construction: technical, geological and environmental risks, among which the hazard occurring in tunnel cross passage construction (TCPC) is a prominent example. In accordance with the aforementioned five steps as seen in Fig. 4, a case study concerning the safety management in TCPC is introduced as follows.

4.1. Step 1: Risk/Hazard identification in TCPC

To ensure the safety of the tunnel fire protection and passenger rescue, the tunnel cross passage is commonly set up between double-tube parallel tunnels, especially for long-distance cross-river tunnels (Zhi et al., 2011). Affected by complex working environments associated with a large buried depth and high water pressure, safety management of TCPC has encountered serious challenges in past years. Although earthwork excavation in TCPC is not a very large project, the structural stability of the main tunnel would be greatly decreased if the tunnel segments are opened. Thus, the excavation work would be likely to produce the abrupt phenomenon of gushing water, basal sand, and even serious casualties. On July 1, 2003, great quantities of sand swamped the tunnel in Shanghai Track Traffic Line Four because of the freezing failure in TCPC, resulting in a serious inclination of an adjacent eight-story building and a collapse of its podium floors. The tunnel was severely damaged and delayed by at least three years, causing a total direct economic loss exceeding US$ 150 million (Zhang, 2003). For the safety related decision analysis, the TCPC located in Wuhan Yangtze Metro Tunnel (WYMT) is chosen to illustrate the application of the fuzzy decision analysis methodology.

WYMT was a double-spool tunnel connecting two large cities, comprising the metropolitan area of Wuhan, Wuchang and Hankou. A slurry shield machine with a cutter diameter of 6.52 m was utilized to push the tunnel from Jiyu Bridge Station to Jianghan Road Station. There were two sets of tunnel cross passages, and the location is shown in Fig. 8. Owing to advantages of strong practicability and environmental protection features, the freezing method had become the first choice in TCPC. Engineering practice indicated that favorable results can be achieved using the freezing method in high water pressure conditions. Frozen ice acted as a bonding agent, fusing together adjacent particles of soil or blocks of rock to increase the combined strength, and make the particles...
impervious to water seepage (Li and Chen, 2012). The freezing method was adopted in TCPC in WYMT, and the vertical section diagram of TCPC is seen in Fig. 9. To be specific, the stability of the retaining structure, which plays a significant role in the system safety of TCPC, was closely related to the whole freezing process. Due to the complicated interaction mechanism in underground environments, the failure was likely to occur at each stage of the freezing process, including the pre-freezing, during-freezing, and post-freezing stages. Next, by reviewing construction standards and technical manuals, and consulting domain experts, relevant risk factors indicating the system safety status and performance, including the pre-freezing failure, freezing failure and post-freezing failure, can be further identified.

4.2. Step 2: Fault tree construction

The TCPC in WYMT was excavated with a cover depth of 42 m and a hydraulic pressure of 0.44 MPa. Under this bad geological condition, the safety of TCPC with the freezing method adopted had a remarkably direct relation with the quality of the freezing effect. During the structure construction process, the boundary constraint conditions and load distribution of the structural system were constantly changing, including the period of frost heaving, excavation, support, and thaw settlement. In the meantime, the connections between the tunnel structure and TCPC’s supporting structure were rigid, increasing the likelihood of the phenomenon of gushing water (sand) on the structural joint surface.

Between 2006 and 2013, researchers at Huazhong Univ. of Science and Technology developed safety control systems for metro construction and operation tasks for Shenyang, Zhengzhou, Shenzhen and Wuhan metro systems. The researchers also developed early warning web-based systems for safety control of each project. A large number of safety related knowledge resources were accumulated during the progress on these projects (Ding and Zhou, 2013; Zhang et al., 2013). On a basis of the previous risk mechanism analysis in TCPC, the potential risk/failure factors and their causal relationships were identified. In the meantime, the Delphi technique was used to reach consensus among different experts’ opinions. First, the risk engineer was interviewed to establish the structure of the fault tree, and then sent to the senior risk coordinator for feedback. Modifications as recommended by the senior risk coordinator were then presented to the risk engineer for further review and feedback. With prior expert knowledge taken into account, the fault logic diagram was subsequently used to build up the failure-consequence scenario from the top to bottom events using logical AND/OR gates. Fig. 10 presents the final fault tree diagram showing the construction safety risk analysis for the retaining structure system of TCPC in WYMT, where 10 basic events contributed to the failure of the top event. The descriptions of all events are illustrated in Table 4.
Subsequently, the frequency assessment of each basic event was calculated, and the average occurrence probability of a specific basic event lying in the ith interval was obtained using Eq. (7). Finally, the “3σ rule” was used to convert the accumulated probabilities into fuzzy failure rates based on Eqs. (8)–(10). The results of fuzzy probability assessment of the aforementioned 10 basic events $x_1 \sim x_{10}$ (see Fig. 10) were presented in Table 5.

4.4. Step 4: Fuzzy based risk analysis

The fuzzy probability of the top event (“the safety of retaining structure”) in TCPC was calculated using Eqs. (2) and (3). Next, at the defuzzification stage, the top-event probability was calculated to be 74.74% using Eq. (14). Matlab7.1 was applied in the programming implementation, and the calculated result was shown in Fig. 13. Herein, the probability curve was made up of massive spots where the precision of cut set $\delta$ reached up to 0.001.

A sensitivity analysis technique was then utilized to evaluate the percentage contribution of the basic events that led to the top event failure in TCPC. A possibility based important index, $FIM$, was deployed for the sensitivity analysis of basic events using Eq. (4). Table 6 presents the calculated results of $FIM$ of basic events. By analyzing these values correctly, more sufficient system components could be selected and improved, and hence the failure probabilities would be reduced. The $FIM$ helps the decision makers to reduce the risk limits by identifying the most critical events, and to work out the risk response strategies.

4.5. Step 5: Consequence analysis and safety management

Basic events with higher importance measures would contribute more to the failure of the top event in TCPC. Therefore, it is important to rank and compare the sensitivity analysis results of all basic events. The ranking results as seen in Fig. 14 illustrate that the most critical basic events to the system failures are related to $x_7$ (Fault in the instability of the frozen curtain), $x_9$ (Fault in floating occurrence in the frozen curtain) and $x_8$ (Fault in poor freezing effect because of inhomogeneity in the frozen curtain). This conclusion once again demonstrates that the quality and strength of the frozen curtain plays a significant role in ensuring the safety of the freezing construction. In addition, poor stability events are prone to occur on the frozen curtain because of excessive convergence deformation caused by partial stress concentration. The results allow us to make the proper arrangements on the failure configuration in order to reduce the risk limit. To guarantee the safety and reliability of the reinforcing scheme in freezing construction, it is necessary to carry out numerical analyses to investigate the distribution of the stress field in the frozen curtain. Furthermore, the strength test should then be conducted based on results of numerical analyses, and the stability safety coefficient should be controlled above 2.0.

From the perspective of the construction period, the top two sensitive basic events both belong to the post-freezing period (see Fig. 14). Therefore, we had reasons to believe that the post-freezing period was the critical sensitive period during the safety management in TCPC, rather than pre-freezing or freezing period. In fact, this deduction was consistent with the actual situation, since the post-freezing period corresponds exactly to the substantive excavation progress. In this period, numerous environmental factors associated with much uncertainty were encountered, contributing to the high sensitivity for the top event failure. Therefore, significant attention should be paid to the displacement and deformation of surrounding soil at this period. The frequency of monitoring should also be increased, ensuring the feedback analysis of measured data in the real time.

![Fig. 9. Vertical section diagram of TCPC in WYMT.](image-url)

4.3. Step 3: Probability assessment

During the expert investigation, questionnaires were administered to a group of experts. A total of 140 questionnaires were distributed, and 115 were returned (the return rate is about 82%), of which 9 were invalid and 106 were valid. Among 106 valid questionnaires, the reviewed experts consisted 64 domain experts with at least five years of working experience, and of 42 research workers in this field. The distribution of the judgment ability level among these experts is illustrated in Fig. 11. Based on the working experience and knowledge background, each expert individually evaluated the occurrence probability intervals of basic events together with their subjective reliability taken into account. The expert confidence index $\theta$ among individuals was then calculated by Eq. (5). In addition, the reliable confidence was distributed among 17 intervals using Eq. (6). For instance, during the occurrence probability assessment for the basic event $x_1$ = “fault in flowing water (sand) while drilling”, one expert with a judgment ability $\zeta = 0.8$ considered the probability interval was within 8th interval, associated with a subjectivity reliability $\psi = 0.7$. That is to say, the most likely occurrence probability for $x_1$ was between 5% and 7% (see Table 2) with a completely reliable confidence $\theta = 0.56$ using Eq. (5). The residual reliable confidence $1 - \theta = 0.44$ was distributed among the other 16 intervals, as seen in Fig. 12. Generally, the expert knowledge was considered as a scarce resource which was not able to provide universal consultation or real-time guidance (Ding et al., 2012). In this way, the data use efficiency can then be highly improved.

![Fig. 10. Fault tree diagram for the safety management of TCPC in WYMT.](image-url)
Table 4
Descriptions of all events in the fault tree diagram.

<table>
<thead>
<tr>
<th>Event code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE</td>
<td>Failure of retaining structure in TCPC</td>
</tr>
<tr>
<td>B₁</td>
<td>Pre-freezing failure</td>
</tr>
<tr>
<td>B₂</td>
<td>Freezing failure</td>
</tr>
<tr>
<td>B₃</td>
<td>Post-freezing failure</td>
</tr>
<tr>
<td>x₁</td>
<td>Fault in the mixture layer of silty clay and fine silt, resulting in flowing water (sand) during the drilling process</td>
</tr>
<tr>
<td>x₂</td>
<td>Fault in the poor welding quality on freezing pipe, resulting in pipe breaks</td>
</tr>
<tr>
<td>x₃</td>
<td>Fault in the precision of drilling equipments, resulting in the extension of freezing time</td>
</tr>
<tr>
<td>x₄</td>
<td>Fault in convergence deformation in the freezing area</td>
</tr>
<tr>
<td>x₅</td>
<td>Fault in poor freezing effect due to inhomogeneity in the frozen curtain</td>
</tr>
<tr>
<td>x₆</td>
<td>Fault in the loss of freezing brine water</td>
</tr>
<tr>
<td>x₇</td>
<td>Fault in the instability of the frozen curtain</td>
</tr>
<tr>
<td>x₈</td>
<td>Fault in fast rate of thawing in the connections between the tunnel structure and TCPC’s supporting structure</td>
</tr>
<tr>
<td>x₉</td>
<td>Fault in the flooding occurrence in the frozen curtain</td>
</tr>
<tr>
<td>x₁₀</td>
<td>Fault in continuous freezing with a long break exceeding 24 h</td>
</tr>
</tbody>
</table>

Fig. 11. Distribution of the expert judgment ability level among valid questionnaires.

Fig. 12. Results of occurrence probability distribution of x₁ obtained from one individual.

Table 5
Results of fuzzy probability assessment of basic events.

<table>
<thead>
<tr>
<th>Basic events</th>
<th>Fuzzy probability assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m - a</td>
</tr>
<tr>
<td>x₁</td>
<td>0.080</td>
</tr>
<tr>
<td>x₂</td>
<td>0.030</td>
</tr>
<tr>
<td>x₃</td>
<td>0.053</td>
</tr>
<tr>
<td>x₄</td>
<td>0.041</td>
</tr>
<tr>
<td>x₅</td>
<td>0.092</td>
</tr>
<tr>
<td>x₆</td>
<td>0.024</td>
</tr>
<tr>
<td>x₇</td>
<td>0.130</td>
</tr>
<tr>
<td>x₈</td>
<td>0.075</td>
</tr>
<tr>
<td>x₉</td>
<td>0.100</td>
</tr>
<tr>
<td>x₁₀</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Fig. 13. Calculated probability of the top event in TCPC.

Table 6
Calculated results of FIM of basic events.

<table>
<thead>
<tr>
<th>Basic event</th>
<th>pₑ (%)</th>
<th>pₑᵢ (%)</th>
<th>FIMᵢ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₁</td>
<td>74.74</td>
<td>71.01</td>
<td>4.99</td>
</tr>
<tr>
<td>x₂</td>
<td>74.74</td>
<td>72.48</td>
<td>3.02</td>
</tr>
<tr>
<td>x₃</td>
<td>74.74</td>
<td>72.23</td>
<td>3.36</td>
</tr>
<tr>
<td>x₄</td>
<td>74.74</td>
<td>73.04</td>
<td>2.27</td>
</tr>
<tr>
<td>x₅</td>
<td>74.74</td>
<td>70.36</td>
<td>5.86</td>
</tr>
<tr>
<td>x₆</td>
<td>74.74</td>
<td>73.78</td>
<td>1.28</td>
</tr>
<tr>
<td>x₇</td>
<td>74.74</td>
<td>65.80</td>
<td>11.96</td>
</tr>
<tr>
<td>x₈</td>
<td>74.74</td>
<td>71.03</td>
<td>4.96</td>
</tr>
<tr>
<td>x₉</td>
<td>74.74</td>
<td>68.27</td>
<td>8.66</td>
</tr>
<tr>
<td>x₁₀</td>
<td>74.74</td>
<td>71.92</td>
<td>3.77</td>
</tr>
</tbody>
</table>

Fig. 14. Ranking results of basic events in fuzzy sensitivity analysis.
5. Conclusions and future research

Metro construction is typically a highly complicated project associated with various potential risks. In recent years, safety management and management of metro construction have attracted broad attention due to their close relation to public safety. Due to the lack of sufficient data, it is difficult to have an exact estimation of the failure rate of the occurrence probability of undesired events, a fuzzy decision analysis method for safety management in tunneling construction has been presented. A typical hazard in TCP such as the one located in Wuhan Yangtze Metro Tunnel is used for a case study. Results demonstrate the feasibility of the proposed method, as well as its application potential.

Also, there are some other projects encountering the similar situation, where the statistical data is insufficient and high potential risks exist in complex environments, such as coal mining, dam monitoring, nuclear power plants and others. Accordingly, during the fuzzy decision analysis, there increases the need for precise failure probabilities for the purpose of safety management in project management practice. To reach the highly required precision for the fuzzy decision analysis, the expert confidence index can be first proposed to ensure the reliability of collected data during the fuzzy probability estimation, with the expert judgment ability and subjectivity being fully considered. Then, an exact approach can be developed for defuzzification based on the representation theorem, attempting to overcome the limitations related to fuzzy linear approximation operations. Furthermore, a possibility based importance index, fuzzy importance measure, can be deployed for the sensitivity analysis of basic events to reveal the critical basic events for reducing the risk limit. This fuzzy decision analysis can be used by industry practitioners and decision support tool to provide guidelines for safety management not only in metro construction, but also in other similar complicated projects.

In this research, the fuzzy decision analysis approach is developed on a basis of the triangular fuzzy numbers. However, in Fuzzy set theory (FST), both trapezoidal and Gaussian fuzzy numbers can also be used to convert the uncertain numbers into fuzzy numbers. Which kind of fuzzy numbers performs to have less information loss during the probability fuzzification and defuzzification process is unknown. Our subsequent research will be focused on the application of the proposed method under conditions of other fuzzy numbers, as well as comparisons among different fuzzification methods.

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References