1) In class it was stated that the spectral density for black-body radiation was given by Planck’s formula: \( \rho(\nu, T) = \left( \frac{8\pi}{c^3} \right) \frac{\nu^2}{e^{\frac{h\nu}{kT}} - 1} \).

a) We argued in class that for low frequencies \((\frac{h\nu}{kT} \ll 1)\) this should reduce to the classical statistical mechanical result of Rayleigh and Jeans----

\[ \rho_{R-J}(\nu, T) = \frac{8\pi \nu^2}{c^3}. \]

Show that it does.

b) Prior to the years immediately preceding the discovery of the Planck formula data for black-body radiation was restricted to relatively high frequencies \((\frac{h\nu}{kT} \gg 1)\) and was found to empirically be fit by Wien’s law \( \rho(\nu, T) = a\nu^3 e^{\frac{h\nu}{kT}} \), where \( a \) and \( b \) are parameters. Starting with the Planck law, find the parameters \( a \) and \( b \) in terms of fundamental constants.

2) Using the conservation of energy and momentum and assuming a two-body collision between a photon and an electron initially at rest derive the Compton formula:

\[ \lambda_s - \lambda_i = \frac{h}{mc} \left( 1 - \cos(\theta) \right) \]

where the subscripts \( i \) and \( s \) indicate the wavelength of the incident and scattered radiation respectively. (Hint: You may want to use relativity to first solve the problem in the center of mass frame and then boost).