AMSC/MATH 420, Spring 2013
Modeling Epidemics: Team Homework 6

due Monday April 15

We'll continue with the two-group SI model with two types of interventions:

$$
\begin{aligned}
dS_1/dt &= -p_{11}S_1\mathcal{I}_1 - p_{12}S_1\mathcal{I}_2 - a_1S_1\\
d\mathcal{I}_1/dt &= p_{11}S_1\mathcal{I}_1 + p_{12}S_1\mathcal{I}_2 - (a_1+b_1)\mathcal{I}_1\\
dS_2/dt &= -p_{21}S_2\mathcal{I}_1 - p_{22}S_2\mathcal{I}_2 - a_2S_2\\
d\mathcal{I}_2/dt &= p_{21}S_2\mathcal{I}_1 + p_{22}S_2\mathcal{I}_2 - (a_2+b_2)\mathcal{I}_2.
\end{aligned}
$$

Let's associate a "cost" $K_c(a_1, a_2, b_1, b_2) = ca_1+ca_2+b_1+b_2$ with an intervention parameter quadruple (a_1, a_2, b_1, b_2), where c is a positive number. Let's also define the "impact" $M(a_1, a_2, b_1, b_2)$ of the quadruple to be the amount by which the intervention reduces the size of the outbreak, as a fraction of the no-intervention outbreak size. So for example, if the outbreak is reduced to 25% of its no-intervention size, then $M = 3/4$, because 3/4 of the people who would have been infected are saved by the intervention.

Set a budget of $K_c(a_1, a_2, b_1, b_2) = 0.04$ and consider the optimal paramaters to be those within the budget that maximize $M(a_1, a_2, b_1, b_2)$. (Note: I'm setting a budget that I think will allow you an impact that is at least around 1/2 no matter what c is. If you are getting impacts very close to 0 or very close to 1 for your transmission parameters, try a different budget.)

If you choose values for 3 of the 4 intervention parameters that are within the budget, you can easily compute the value of the 4th parameter that uses up the entire budget, and then compute the impact of the resulting parameter quadruple; this reduces the problem of finding the optimal parameters to maximizing a function of 3 variables. This is few enough variables that you can probably search over the entire set of (nonnegative) parameters that are within the budget.

For each of the two metropolitan areas you were assigned, use the transmission parameters $p_{11}, p_{12}, p_{21}, p_{22}$ you found by fitting the data and answer the following questions:

1. What is the largest value of c for which the optimal parameters have $b_1 = b_2 = 0$? (One decimal place of accuracy is fine for this and the next question.)

2. What is the smallest value of c for which the optimal parameters have $a_1 = a_2 = 0$?

3. For a range of c values in between the values you found above, determine the optimal a_1, a_2, b_1, b_2 and graph these values as a function of c. Are there values of c for which the optimal parameters are all positive, and/or for which 3 of 4 are positive?