Ionization: what makes a plasma

Supply energy to a neutral gas. What determines the degree of ionization?

Important:

Two processes:

Electron impact ionization:

Before:

\[ \text{Atom} \rightarrow \text{ion} + \text{two e}^- \]

After:

Photo ionization:

\[ \text{Photon} \rightarrow \text{ion} + \text{electron} \]
Inverse processes

Before

Three body recombination

Ion \rightarrow Electric \rightarrow Ion

After

Atm

Radiative recombination

Electron \rightarrow Photon \rightarrow Atm
In thermal equilibrium these processes balance and determine the fraction of atoms that are ionized (consider H for simplicity)

\[ N_i = \text{density of ions} \]

\[ n_e = \text{density of electrons} \]

fraction of ionized

\[ f = \frac{N_i}{n_i + n_e} = f(n_i, T) \]

depends on T & density

Plasmas in Equilibrium are rare.

Photons are not confined

\[ \text{Photons} \]

\[ \text{Plasma} \]
Executive Summary

1. For $T \lesssim 1 \text{ eV}$ almost all atoms are ionized $E_{\text{ion}} = 13.6 \text{ eV}$
   we'll see why

2. Unless $T \lesssim \text{ several } \text{ eV}$ and
   $n \leq 10^{16} \text{ cm}^{-3}$

Radiative Recombination > Three body
proportional to $n^2$ $T^3$
(assumes protons not confined)
No photon continuum
Coronal Equilibrium

$\frac{N_I}{N_{\text{tot}}} = g(T) < \text{fluct}) \text{ depends only on temp.}$

Radiative Processes see The
"Principles of Plasma Spectroscopy"
Hans R. Griem
Cambridge University Press,
Reaction Cross-Sections

Various atomic and nuclear processes are described by reaction rates and cross-sections.

What does this mean?

Suppose we have an

* To be concrete let's consider an example, collisional ionization

We would like to know the rate at which new electrons are ejected in a plasma by collisions of existing free electrons by recombination with neutral atoms.
The rate at which an atom is ionized is given by

\[ \sigma \nu = \frac{\text{# of ionizations}}{\text{sec}} \]

\( \sigma \) depends on the energy of the beam, \( \nu^3 \frac{1}{2} m \nu^2 \)

Note: \( \sigma \) has units of area

\[ \text{Area} \times \frac{\text{# of electrons}}{\text{Area s e c}} = \frac{\text{#}}{\text{sec}} \]

\( \sigma \) = ionization cross-section

This is the rate if we have an incident beam with velocity \( \nu \). For a distribution of electrons, the rate at which the atom is ionized is

\[ \langle \sigma \nu \rangle = \int d^3 \nu \int_0^\infty f_e(\nu, \nu_{\text{speed}}) \sigma \nu \]
First, the basic process is characterized by a cross quantity called the cross-section. The cross-section represents the size of the target.

Suppose we have an infinite beam of electrons with speed $v$.

$\# \text{ sec} = FA$

![Diagram of a neutral atom.

The flux of electrons is $F = N eV$.

$F = \frac{\# \text{ of electrons}}{\text{cm}^2 \text{ sec}}$

The rate at which each electron encounters a single neutral atom.

Assume that each electron encounters the same neutral atom.

Some electrons miss the atom.

Some electrons excite the electron to a higher state but do not ionize.

Some electrons ionize the atom.
\( E_H = \frac{e^2}{2a_0} \) \text{ eV/s} \quad E_H \sim 13.6 \text{ eV}

\( E_\infty = \text{energy needed to ionize} = E_H \text{ in hydrogen} \)

\( \Delta \sim \frac{1}{E} \quad \text{for high energy} \)

Other ionization processes:
- Photo ionization
- Knock-up

Ionization rate

\[ \frac{\text{d}n_e}{\text{d}t} = n_a \langle \sigma v \rangle \]

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\[ V_i \]

\[ V_i = \text{ionization rate} \times n_e \]

function of \( T_e \)

\[ \sim V_i \sim V_e \text{ collision} \]
Note this is proportional to electron density and depends on electron plasma electron temperature.

Finally, the number of ionizations per unit volume is obtained by multiplying by the atomic density.

\[ n_e = n_a \langle E_{\text{enV}} \rangle \]

But what is \( E \)?

**Thomson calculation**

\[ a_0 = \frac{\hbar^2}{m e^2} \]

Electron scatters from rest if it picks up enough energy it is free.

\[ \sigma = 4\pi a_0^2 \left( \frac{E_{\text{enV}}}{E} \right)^2 \left( \frac{E}{E_0} - 1 \right) \]
Recombination

be fore

after

Three body recombination

\[
\frac{\text{d}n_e}{\text{d}t} = n_a V_I - n_i V_R
\]

\( V_R \) is the recombination rate

depends on \( n_e, T_e \)

Principle of detailed balance

In thermal equilibrium rate of

ionization = rate of recombination

\[
V_R = \left( \frac{n_a}{n_i} \right) V_I
\]

we call this last lecture

where \( \left( \frac{n_a}{n_i} \right) \) = ratio of neutrals to ions in TB
\[ \frac{\text{d}n_e}{\text{d}t} = n_a \nu_I - n_e \nu_F \pm 8 \pi^{3/2} a_0^3 n_T \left( \frac{E_k}{T} \right)^{3/2} e^{-\frac{E_k}{T}} \]

- Three powers of density
- Two powers of density
Thermal equilibrium (Saha Equilibrium)

Suppose we have \( N_T = N_a + N_i \) atoms

\( N_a = \text{number of neutral atoms} \)

\( N_i = \text{number of ions} \)

Consider case of hydrogen singly ionized only (assume molecules \( \text{H}_2 \) dissociated)

\( ; \text{At a given temperature and density} \)

\( \text{what fraction of atoms are ionized?} \)

* bottom line –

* probability that system is in state of energy \( E \) is proportional to \( \exp(-E/\Theta) \)

\[ \text{Consider box containing 1 atom} \]

\[ \text{side } L = N_T^{-4/3} \]
Bottom line

\[
\left( \frac{n_\Sigma}{n_T} \right)_{th} = \frac{1}{8\pi^{3/2} a_0^3 n_T \left( \frac{E_h}{T} \right)^{3/2} e E_h/T + 1}
\]

(Borel radius) total density

when \( E_h n_T \uparrow \frac{a_0^3 n_T}{T} \ll 1 \)

dilute gas

almost

all atoms are confined

\[ E_h = 13.6 \text{ eV} \]

with temperature fixed \( n_T \uparrow \frac{n_\Sigma}{n_T} \downarrow \)

with density fixed \( T \uparrow \frac{n_\Sigma}{n_T} \rightarrow 1 \)
Ionization Fraction

\[
\left( \frac{n_i}{n_T} \right)_{th} = \frac{\sum_{\text{states } E > 0} e^{-\frac{E}{kT}}}{\sum_{\text{states } E \leq 0} e^{-\frac{E}{kT}}} + \sum_{\text{states } E > 0} e^{-\frac{E}{kT}} \]

Let's evaluate states \( E > 0 \), neglect atomic potential (assume plasma parameter is small)

Box of size \( L \)

\[
\psi \sim e^{ik \cdot x}
\]

Wave number
\[
k = \frac{2\pi}{L} \left( \frac{e_x}{x} + \frac{e_y}{y} + \frac{e_z}{z} \right)
\]

\( n_{x,y,z} = -\infty \rightarrow 0 \rightarrow \infty \)

\( p = \hbar k \)

\[
E = \frac{\hbar^2 k^2}{2m}
\]

\[
\sum_{\text{states } E > 0} e^{-\frac{E}{kT}} = \sum_{n_x, n_y, n_z} \exp \left[ -\left( \frac{\hbar}{L} \right)^2 \frac{1}{2mT} (n_x^2 + n_y^2 + n_z^2) \right]
\]

For large box \( \sum_{n_x, n_y, n_z} = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \)
\[ \sum e^{-E/kT} = \left[ \int d\nu \exp \left[ -\left( \frac{\hbar}{L} \right)^2 \frac{\nu^2}{\hbar^2} \right] \right]^3 \]

\[ = \left( 2\pi m^2 \right)^{3/2} \left( \frac{L}{\hbar} \right)^3 \]

\[ = \frac{L^3}{\hbar^3} \left( \frac{2\pi m \hbar}{\hbar^2} \right)^{3/2} \]

Bohr radius \( a_0 = \frac{\hbar^2}{me^2} \)

\[ E_{1+} = \frac{e^2}{2a_0} = 13.6 \text{ eV} \]

\[ \hbar^2 = a_0 \frac{me^2}{e^2} \]

\[ \hbar^2 = m \frac{2a_0 E_{1+}}{E_{1+}} \]

\[ n = \left( 2mE_{1+} \right)^{1/2} a_0 \]

\[ \hbar = 2\pi \hbar \]

\[ \sum e^{-E/kT} = \frac{L^3}{(2\pi m \hbar)^{3/2}} \left( \frac{2\pi m \hbar}{\hbar^2} \right)^{3/2} = \frac{L^3}{a_o^3} \frac{\pi^{3/2}}{2 \pi^{3/2}} \left( \frac{E_{1+}}{E_{1+}} \right) \]

Now put \[ \frac{L^3}{\hbar} \]
\[
\sum_{E > 0} e^{-\frac{E}{T}} = \sqrt{\frac{1}{\alpha_0^3 n_r}} \frac{1}{8\pi^{3/2}} \left( \frac{T}{E_h} \right)^{3/2}
\]

very big factor for dilute gas

what that means is if \( T \sim E_h \)

almost all atoms are ionized

(much more phase space \( \sim \) volume available to free electrons)

result for a dilute gas is comparable

fractions of neutral and ionized atoms occur for \( E_h / T \gg T \)

\[ \sum_{E > 0} e^{-\frac{E}{T}} \]

is dominated by ground state

\[ \sum_{E > 0} e^{-\frac{E}{T}} = e^{-\frac{E_h}{T}} \]
\[
\left( \frac{n_i}{n_T} \right)_{TH} = \frac{1}{a_0^3 n_T} \frac{1}{8 \pi^{3/2}} \left( \frac{T}{E_H} \right)^{3/2} \exp \left( \frac{E_H l T}{T} \right) \left( \frac{T}{E_H} \right)^{3/2} \frac{1}{a_0^3 n_T} \frac{1}{8 \pi^{3/2}} \left( \frac{T}{E_H} \right)^{3/2} \\
(\frac{n_a}{n_i})_{th} = \frac{e}{8 \pi^{3/2} a_0^3 n_T} \left( \frac{T}{E_H} \right)^{3/2} n_T \propto n_T \sim n_e \\
\frac{dn_e}{dt} = n_e V_x - n_i \left( \frac{n_a}{n_i} \right) V_x \\
\text{three body process} \\
50\% ~ \text{ionized} \quad \left( \frac{E_H}{T} \right)^{3/2} \exp \left( \frac{E_H}{T} \right) = \frac{1}{8 \pi^{3/2} a_0^3 n_T} \\
\text{See picture} \]