Collisions in Plasmas

General Comments

1. Collisions are not like colliding Billiard Balls
   - Billiard Balls - collisions occur one ball at a time

- Large angle deflection

- Plasma Particles - a given particle is colliding with all the other particles within a Debye sphere (Coulomb Force is long range)

- Many small angle deflection diffusion and slowing down in velocity space

△
2. Due to disparity in mass $M_i \gg m_e$ collision occur at different rates for ions and electrons, different collision processes occur at different rates

$Zve \rightarrow Ve \rightarrow V_i \rightarrow Veq$

\[
\frac{1}{(M_{i}\text{time})^2} \frac{1}{(m_e\text{time})^2}
\]

average conversion stage

1. Electrons isotropize $fe \rightarrow f_e(\text{IV1})$

2. Electrons thermalize with helium $f_e(\text{IV1}) \rightarrow \frac{N}{(2\pi\hbar m_e)^{3/2}} \exp(-\frac{1}{2}m_e\text{v}^2/\hbar^2)$

3. Ion thermalize $\frac{f_i(\text{V})}{f_i(\text{IV})} \rightarrow \frac{N}{(2\pi\hbar m_i)^{3/2}} \exp(-\frac{1}{2}m_i\text{v}^2/\hbar^2)$

4. Electron and ion equilibrium $T_e = T_i$

$V_e \sim \frac{D_n}{V^2}$ diffusion constant

$\lambda = \frac{W_0}{n_i \nu^2}$ length

$V \sim \frac{W_0}{n_i \nu^3}$
Where does the $Z$ come from?

$$Z e / 0 \quad 0 / Z e$$

for equal total am m.

$$Z e / 0 \quad 0 / Z e$$

Putting more charge in each scatter even though there are fewer scatters results in higher diffuseness. Think in term of random walk in angle.

**Step size $\Theta \sim Z$**

**Time between collision $\tau \sim Z$**

$$D \sim \frac{\Delta \Theta^2}{\tau} \sim Z$$

Collisions are elastic.

Where does the factor $(m_1/m_2)^{1/2}$ come from?

Suppose for the moment the scatters are fixed.

$$\frac{d}{dt} e \frac{d}{dt} = 9 e B E(x(t))$$

let $\nu = \lambda \gamma$,

$$\gamma \cdot \gamma = t / \gamma$$

$$\frac{d}{dt} = \nu \frac{dx}{dt} = \lambda^2 \gamma^2 B_0$$

$$\frac{d}{dt} = \lambda \gamma \sqrt{\frac{\gamma}{\gamma}}$$
Equilibration can be studied ball electron vs. ping-pong balls
to lowest order there is no energy exchange in a collision

From momentum conservation

\[ m_e \Delta V_e = -m_i \Delta V_i \]

Hence

\[ \frac{\langle \Delta V_i \rangle^2}{\tau} = \frac{m_i^2 \langle \Delta V_i^2 \rangle}{m_i^2} \cdot \frac{m_i \langle \Delta V_i \rangle^2}{\tau} \approx \frac{m_e \langle V_e \rangle^2}{m_i^2} \]

No rate of exchange in energy \( \sim \frac{\text{rate of elastic}}{m_i} \) either
Collisions  Using  Vlasov Eqn

\[ F_N \left( x_1, y_1, x_2, y_2, \ldots, t \right) = F_1 \left( x_1, y_1, t \right) F_1 \left( x_2, y_2, t \right) \]

Assumption of iid eliminates detailed effects of individual particles on each other, i.e., it eliminates collisions.

Collisions may still be important over long time scales.

To estimate effect of collisions consider effect of one stationary charge on another moving charge.

\[ \delta V = \delta t \cdot \frac{eE}{m} \]

\[ \delta t = \text{time during which Field is felt} \]

\[ \delta t \approx \frac{r_0}{V} \quad \frac{E}{V} \approx \frac{e}{r_0^2} \]

\[ \delta V = \frac{e^2}{r_0^2} \]


Let's construct a diffusion coefficient in velocity space using random walk arguments.

\[ \frac{\partial f}{\partial t} = D \nabla^2_v f \]

\[ D = \text{velocity space diffusion coefficient} \]

\[ D = \frac{(8V)^2}{\Delta T} \quad 8V = \text{deflection in a collision} \]

\[ \Delta T = \text{time between collisions} \]

\[ 8V = \frac{e^2}{mvf_0} \quad \text{collision is when electron passes within } r_0 \text{ of scatterer} \]

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\[ L = \Delta T \]

We can expect another collision when volume = \( IV_0^2L = \frac{1}{\nu} \).
\[ L = \frac{1}{\pi n r_0^2} \]
\[ \Delta T = \frac{1}{n \pi n r_0^2} \]

\[ \frac{(\delta V)^2}{\Delta T} = \frac{\pi e^4 n n r_0^2 V}{m^2 V^2 r_0^2} = \frac{\pi e^4 n}{V m^2} = D \]

Notice \( n \) disappears

\[ \frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \left[ D \frac{\partial f}{\partial V} \right] \]

"Collision frequency"

\[ (D/V^2) = \frac{\pi e^4 n}{m^2 V^3} \]

\[ V \propto n \propto V^{-3} \]

Let try to be more careful
Let's sum up the effects of collisions occurring with different $v_0$

\[ D = \sum \frac{\frac{\pi e^4}{m^2 v^2 r_0^2 \Delta T(r_0)}}{r_0} \]

\[ D = \sum \frac{8V(r_0)}{\Delta T(r_0)} \]

$T(r_0) =$ Time between collisions with a pattern particular $r_0$

Volume $= \pi r_0 dr_0 L(r_0) = \frac{1}{\pi} L(r_0) = \frac{\Delta T(r_0) \pi r_0}{\pi}$

\[ \frac{1}{\Delta T(r_0)} = \frac{8V}{L(r_0)} = \pi r_0 dN V \]

\[ \frac{8V(r_0)}{\pi r_0} = \frac{e^2}{m^2 v} \]

\[ D = \int \frac{e^4}{m^2 v^2 r_0^2} \pi r_0 dN v = \int \frac{\pi e^4}{m^2 v} \frac{dN}{r_0} \]

Integral does not converge.
Estimate of $D_{vv}$

$$D_{vv} = \int_{r_0}^{\infty} \frac{d(\sigma)}{d\tau} \frac{n_0 e^4}{r_0^2 m^2 v}$$

$$\frac{1}{\Delta t} = n_0 \sigma_0 n_0 v$$

$$\Rightarrow D_{\sigma \sigma} = \int n_0 \sigma_0 n_0 v (\sigma V(n_0))^2$$

$$\sigma V(n_0) = \frac{e^2}{mv_0 r_0}$$

$$\text{estimate}$$

$$\begin{align*}
D_{\sigma \sigma} &= \int_{r_{min}}^{r_{max}} n_0 \sigma_0 n_0 v \frac{e^2}{m^2 V r_{min}^2} = \frac{n_0 e^2}{\pi n^2 V} \frac{1}{\Delta t} \\
D_{\sigma \sigma} &= \int_{r_{min}}^{r_{max}} n_0 \sigma_0 n_0 v \frac{e^2}{m^2 V r_{min}^2} = \pi n e^2 \ln \left( \frac{r_{max}}{r_{min}} \right)
\end{align*}$$
\[ D = \frac{TVa}{E} \ln \frac{\nu_{\text{max}}}{\nu_{\text{min}}} \]

We assumed small deflections, and this is easy as \( \nu_{\text{min}} \) is very large.

\[ \phi = \frac{e^2}{\varepsilon_0} \]

Force gets weaker as \( \nu \to \infty \).

\[ \phi(\nu) = \frac{e^2}{\varepsilon_0} \]

\[ 5\nu = \frac{e^2}{\varepsilon_0 \nu} \]

\[ \nu_{\text{min}} = \frac{MV^2}{e^2} \]

\[ \nu_{\text{max}} = \frac{MV^2}{e^2} \]

Cross-section approximation.
other particles will shield

\[ \lambda_d = \frac{1}{4\pi \varepsilon_0 e^2} \]

\[ r > \lambda_d \]

Field goes to zero

\[ r_{\text{max}} = \lambda_d \]

\[ r_{\text{min}} = \frac{1}{2} \frac{e^2}{2mV^2} \]

\[ \ln \left| \frac{r_{\text{max}}}{r_{\text{min}}} \right| = \frac{1}{2} \ln \left( \frac{\lambda_d^2}{r_{\text{min}}} \right) = \frac{1}{2} \ln \left( \frac{T}{4\pi e^2} \right) \]

\[ \frac{r_{\text{min}}}{\lambda_d} = \frac{4\pi e^2 n}{T} \frac{1}{4\pi n} = \frac{4\pi n \lambda_d^3}{3} \]

= \frac{1}{\# \text{ particles in debye sphere}}
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\[ \frac{r_{\text{min}}}{\lambda_d} = \frac{4\pi e^2 n}{T} \frac{1}{4\pi n} = \frac{4\pi}{3} \frac{\lambda_d^3}{\lambda_d} \]

\[ = \frac{1}{\# \text{ particles in Debye sphere}} \]
\[
\frac{r_{\text{max}}}{r_{\text{min}}} \approx \frac{\lambda_d}{e^2/\text{m} \cdot \text{v}^2} \approx \frac{\lambda_d \cdot T}{e^2} \times \frac{4\pi n}{4\pi n} = \frac{\lambda_d}{e^2} \cdot \frac{T}{n} \approx 4\pi n \lambda_d^3
\]

If # of particles is large, small angle collisions dominate.