1. A charged particle drifts in an electric and magnetic field. The magnetic field is due to an infinitely long wire carrying a current along the z axis. The electric field is uniform and directed along the z axis. (This is essentially Goldston and Rutherford #3.8)

   a) Write the equations for the motion of the guiding center in the R-z plane.

   b) Write an equation for the rate of change of kinetic energy of the particle. (Recall the particle is moving in a constant electric field).

   c) Show from a and b that the longitudinal action \( J = \int \Phi \, dl \, m v_\parallel \) is a constant of motion.

   d) Solve for the time dependence of the coordinates of the guiding center and the magnitude of the parallel and perpendicular velocities of the particle. (For example, \( R(t) = R(0) \exp[-\gamma t] \) where \( \gamma = c E_z / BR \) is a constant.)

2. Imagine that you have an isotropic magnetized plasma with \( T_\parallel(0) = T_\perp(0) = T_0 \). The distribution function is given by,

   \[
   f = n_0 \left( \frac{2\pi m}{\varepsilon} \right)^{\frac{3}{2}} T_\parallel^{-1/2} T_\perp^{-1} \exp \left( -\frac{m v_\parallel^2}{2T_\parallel} - \frac{m v_\perp^2}{2T_\perp} \right).
   \]

   The magnetic field is increased in time to double its initial value. The rate of increase is slow compared with the gyrofrequency of particles, but fast compared with the rate at which collisions establish isotropy.

   a) Find the new parallel and perpendicular temperatures.

   Now let the temperatures equilibrate due to energy conserving collisions. Assume that the plasma is isolated from the external world.

   b) Find the isotropic temperature.

   Now decrease the magnetic field to its initial value, again at a rate that is slow compared with the gyrofrequency, but fast compared with the collision frequency.

   c) Find the new parallel and perpendicular temperatures.

   Let the temperatures equilibrate,

   d) What is the final temperature?

   e) How would your answer differ if the rate of increase in magnetic field had been much slower than the collision rate? What is the source of energy that accounts for the temperature rise in the first case (known as magnetic pumping)? (This is problem 4.2 of Goldston and Rutherford)

3. Electrons and ions move in a uniformly magnetized \( B = B_0 \hat{a}_z \) plasma. The are subjected to a low frequency \( \omega \ll \Omega_i, \Omega_e \) electric field of the form \( E = \text{Re}\{ \hat{E} \exp[i k \cdot x - i \omega t] \} \). Using the
E×B and polarization drifts for motion transverse to the magnetic field derive an expression for the dielectric tensor. Show that it is equivalent to the one derived in class in the appropriate limit.