1. Show that if one assumes a distribution function of the form:

\[ f(x,v,t) = \frac{n}{(2\pi T/m)^{3/2}} \exp \left[ -\frac{m(v-u)^2}{2T} \right], \]

where \( n, T, \) and \( u \) are functions of \( x \) and \( t \), then the first three moments of the Vlasov equation reproduce the equations governing an ideal fluid. Interpret the equation of state.

2. Show that

\[ f(x,v,t) = n(x,t)\delta(v-u(x,t)), \]

where \( \delta(v-u(x,t)) \) is a three dimensional delta function satisfies the Vlasov equation. What are the conditions on \( n(x,t) \) and \( u(x,t) \)? Interpret this in light of your result for problem #1. You may need to use the identity

\[ g(x) \frac{d}{dx} \delta(x-x_0) = g(x_0) \frac{d}{dx} \delta(x-x_0) - \delta(x-x_0) \frac{dg(x_0)}{dx_0}. \]

3. Show that \( f = f(v_z,v_x^2,v_y^2,X,Y) \) is a steady solution of the Vlasov equation in a uniform magnetic field \( B(x) = B\hat{z} \). Here \( X = x + v_y/\Omega, \) and \( Y = y - v_x/\Omega \) where \( \Omega = qB/mc \). Interpret \( X \) and \( Y \) in terms of the motion of individual particles.

4. A cylindrically symmetric plasma \((r, \theta, z)\) has a distribution function given by:

\[ f(x,v,t) = \frac{n}{(2\pi T/m)^{3/2}} \exp \left[ -\frac{H - \lambda P_{\theta}}{T} \right], \]

where \( H \) is the energy and \( P_{\theta} \) is the canonical angular momentum, \( P_{\theta} = r(mv_{\theta} + qA_{\theta}/c) \). The quantities \( n, \lambda, \) and \( T \) are constants. Let the vector potential \( A_{\theta}(r) \) produce a uniform magnetic field. Find expressions for the density and rotation rate as a function of \( r \).