Physics 606: Homework #3

Due: Tuesday March 6, 2018

Jackson: Problems 3.5, 4.8, 4.9, 4.10, 4.11 – not graded

1. An infinite length line charge of density $\rho$, [C/m] lies parallel to the z-axis and passes through the point $(x = x_0, y = y_0, z = 0)$. Surrounding the line charge is a conducting cylindrical shell of radius $a > \sqrt{x_0^2 + y_0^2}$.

(A) Draw by hand your best guess of the level curves of constant potential and electric field lines inside the cylindrical shell, $a > x^2 + y^2$.

(B) Now derive a mathematical expression for the potential inside the cylinder.

(C) What is the force per unit length on the line charge, and where must the line charge be for the force per unit length to vanish?

(D) What is the total linear surface charge density on the inside wall of the cylinder [C/m].

2. A large slab of dielectric material with dielectric constant $\varepsilon > \varepsilon_0$ (Let’s call it Jarlsberg cheese) is immersed in a uniform electric field $E = E_0 \hat{z}$. Inside the material there is a spherical void of radius $a$, which distorts the electric field in the vicinity of the void.

(A) Obtain expressions for the following quantities inside and outside the void, $\Phi$, $E$, $D$, $P$ (Take the potential to be zero at the center of the void.)

(B) Indicate on a drawing the location of any induced surface charge and obtain an expression for the charge density.

(C) Now add a point charge $q$ at the center of the sphere, how do your answers to the above change?

3. For the model frequency dependent dielectric constant introduced in class:

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2},$$

show that the expression

$$W = \frac{1}{4} \int d^3x \frac{\partial \omega \varepsilon(\omega)}{\partial \omega} |\hat{\mathbf{E}}|^2$$
includes three contributions to stored energy. Identify each of the three modes where energy is stored.