Two dimensional solutions of Laplace's equations in Cartesian coordinates are easy to come by. Let \( z = x + iy \) be a complex number, and \( f(z) \) any complex analytic function of \( z \). Examples of analytic functions are: \( z^n, \sin z, e^z \ldots \) The complex function \( f \) will have a real part \( f_R(x, y) \) and an imaginary part \( f_I(x, y) \) each of which depend on \( x \) and \( y \). Both \( f_R \) and \( f_I \) can be regarded as the real functions of \( x \) and \( y \) as well as the real and imaginary parts of the complex function \( f = f_R + i f_I \).

Show that \( f_R \) and \( f_I \) are both solutions of Laplace's Equation. Along the way you must first show the following (Cauchy-Riemann) equations,

\[
\frac{\partial f_R}{\partial x} = \frac{\partial f_I}{\partial y}, \quad \frac{\partial f_I}{\partial x} = -\frac{\partial f_R}{\partial y}.
\]

Take \( f(z) \) to be \( \arcsin(z) \). Make contour plots of the potential corresponding to the real part of \( f \). What problem is this the potential for?

2.B Point charges \( q_1 \) and \( q_2 \) are located at the points \((x_1, 0, 0)\) and \((0, y_2, 0)\) respectively. A perfectly conducting sphere of radius \( a < |x_1|, |y_2| \) is located at the origin. The sphere is held at a potential of \( V \) volts. Find a) the potential outside the sphere, b) the total charge on the sphere, and c) the force on the sphere.

2.C A line charge \( \rho_L \) is located parallel to and above a conducting plane (the \( y-z \) plane) passing through the point \((x = d, y = 0)\).

i. Obtain an expression for the potential as a function of \( x \) and \( y \).
ii. Make contour plots of lines of constant potential in the \( x-y \) plane.
iii. Show that the potential has a constant value \( V \) on a cylindrical surface with axis passing through the point \( x = x_c \) and \( y = 0 \) and radius \( r_c \) where \( r_c = d / \sinh \Gamma \) and \( x_c = d \coth \Gamma \) and \( \Gamma = 2\pi \epsilon_0 V / \rho_L \).
iv. Use the result of C to determine the capacitance per unit length of a parallel conductor transmission line, where the line has two cylindrical conductors of radius \( r_c \), and centers separated by \( s \).