1. (50 pts) A line charge \((K \text{ Coul/m})\) lies parallel to and above a conducting surface. The surface is in the form of a plane with a semi-cylindrical ridge of radius \(a\). The line charge is located with respect to the axis of the semi-cylindrical ridge by the cylindrical coordinates \((r_0, \theta_0)\). See the picture below.

![Diagram of line charge and conducting surface with semi-cylindrical ridge](image)

a) Use separation of variables and orthogonal functions to find the potential throughout space.

b) Obtain an expression for the surface charge density and the total surface charge per unit length on the semi-circular ridge.

c) If \(\theta_0 = \pi / 2\) and \(r_0 >> a\), what is the value of the electric field on the top of the ridge \((\theta = \pi / 2)\)? How does this compare with the value you expect if there were no ridge at all?

d) Is there a way to solve this problem using image charges alone? Explain.
2. (25 pts) A spherically symmetric volume charge density \( \rho(r) \) exists between two spherically symmetric conducting surfaces at \( r = a \) and \( r = b > a \). The potential satisfies the following boundary conditions at the two surfaces, \( \phi(a) = 0 \) and \( \phi(b) = V_0 \). Show that the potential for \( a < r < b \) is given by

\[
\phi(r) = \frac{ab}{b-a} \left[ V_0 \left( \frac{1}{a} - \frac{1}{r} \right) + \int_a^b \frac{r'^2}{\varepsilon_0} \rho(r') \left( \frac{1}{a} - \frac{1}{r} \right) \left( \frac{1}{r} - \frac{1}{b} \right) dr' \right]
\]

Based on the above, what is the Dirichlet Green’s function for this problem?

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3. (25 pts) A disk of charge Q and radius b lies in the x-y plane. Find an expression for the potential in all space in terms of spherical harmonics. (Separate expressions apply depending on whether \( r \) is greater or less than b.) Find the dipole and quadrupole moments of the charge distribution.