ENEE681: Homework #1

Due: Tuesday Feb. 10, 2020

Zangwill: Problems (not graded) 14.2 (find an expression for the voltage across the gap), 14.6, 14.7, 14.22, 15.1

1) A system of oscillating free charge and current density creates an electric field, which after all transients have decayed is of the form

\[ E(x,t) = e_x E_0 \exp(-\sigma r^2) \cos(kz - \omega t) \]

where \( \sigma, k, \) and \( \omega \) are specified parameters, and \( r^2 = x^2 + y^2 \). Assume the medium is described by linear constitutive relations involving \( \mu = \mu_0 \) and \( \epsilon = \epsilon(\omega) \) where \( \epsilon(\omega) \) is a known function of frequency.

a) Find the corresponding magnetic field intensity, \( H(x,t) \).

b) Plot the phasor amplitude at \( z=0 \) of the \( x, y \) and \( z \) components of \( E \) and \( H \) as functions of \( x \) (for \( y=0 \)) and \( y \) (for \( x=0 \)).

c) Take the limit \( \frac{\sigma}{k^2} \ll 1 \) (This assumption applies to the remainder of this problem), and find the free current density that is creating these fields. For what value of \( \omega \) is the current density very small, explain your result.

d) Assume \( \epsilon \) is real and calculate the average energy per unit length in \( z \) associated with the i) electric and ii) magnetic fields. iii) What is the power flowing through the plane \( z=0 \)?

e) Now assume that \( \epsilon = \epsilon_r + i\epsilon_i \), and calculate the power per unit length in \( z \) transferred from the free current to the fields. Where does this power go?

f) Suppose there is no free current density; your answer to part c determines the allowed value of \( \omega \). What happens if \( \epsilon = \epsilon_r + i\epsilon_i \) is complex? Derive an expression for the value of \( \omega \) in the case \( \epsilon_r \gg \epsilon_i \). Interpret your results in terms of your answers to parts d and e.

2) Consider a pair of coaxial conductors as shown in the figure. The outer conductor has infinite conductivity, while the inner conductor has conductivity \( \sigma \). First consider the case \( b=a \) and derive an equation for evolution of the current density in the direction along the axis of the inner conductor (neglect Maxwell's displacement current). Find the normal mode solutions for symmetric (\( j_z \) only depends on \( r \)) current densities. Now suppose \( b>a \). Show that under certain approximations the total current in the inner conductor satisfies
\[
\frac{dl}{dt} = -\frac{2}{a^2 \mu_0 \sigma \ln(b/a)} I.
\]