1. Under some conditions waves can propagate along the surface separating two media as illustrated schematically in the figure below. These waves have sinusoidal dependence on time and direction along the surface, in this case \( z \). The fields decay exponentially with distance from the surface, in this case \( x \).

a) Derive the conditions necessary for this kind of wave to exist. Be sure to consider the polarization of the fields. There are two choices just as in the case of waves incident on an interface and the conditions are different for the two polarizations.

b) In the case in which \( \mu_\uparrow = \mu_\downarrow = \mu_0 \), \( \varepsilon_\uparrow = \varepsilon_0 \), and \( \varepsilon_\downarrow = \varepsilon_0 \left(1 - \omega_p^2 / \omega^2 \right) \) derive and plot the dispersion relation, \( \omega \) versus \( k \).
2. A charged particle moving in vacuum has the trajectory,

\[ z(t) = vt, \]
\[ x(t) = \begin{cases} a(\cos \Omega t - 1) & 0 < t < 2\pi m / \Omega \\ 0 & \text{otherwise} \end{cases}, \]

and

\[ y(t) = 0. \]

What restrictions should be placed on \( \Omega, a, \) and \( v? \)
Calculate the energy radiated per unit frequency and per unit solid angle in the + and \( -z \) directions. (Bonus: calculate for all directions)
3. Consider the motion of a charged particle in crossed uniform electric and magnetic fields, \( \mathbf{E} = (E_0, 0, 0) \), \( \mathbf{B} = (0, 0, B_0) \).

A) Find a magnetic vector potential such that the Hamiltonian depends only on one spatial coordinate. Write the Hamiltonian and indicate what are the constants of motion.

B) Suppose the particle passes through \( x=0 \) with momentum \( p_{x0} \), and momentum components in the y and z directions both zero. What is the value of the Hamiltonian? (take the potential to satisfy \( \Phi(x=0) = 0 \). What are the locations where the particle will eventually turn in x (\( p_x = 0 \))? What restrictions on \( E_0 \) and \( B_0 \) apply for this to happen?

C) Find a transformation to a moving frame where either the electric or magnetic fields are zero. Use this to interpret your answer to B).