1. (45 pts) Plane waves propagate in a gyro-tropic medium, i.e. one that has a dielectric tensor in the form:

\[
\begin{bmatrix}
\varepsilon_\perp & i\varepsilon_x & 0 \\
-i\varepsilon_x & \varepsilon_\perp & 0 \\
0 & 0 & \varepsilon_\parallel
\end{bmatrix}
\]

where all the \(\varepsilon\)'s are real.

A) For the special case of propagation in the “||” (that is in the z) direction derive a dispersion relation for the different polarizations of electromagnetic waves.

B) Suppose a wave is linearly polarized at \(z=0\). Show that if the wave is linearly polarized at \(z=0\), it will be linearly polarized for all \(z\), but with a polarization direction that varies with \(z\).

C) Obtain an expression for the rate of variation.
2. (60 pts) Electromagnetic waves propagate in a medium (with constant 
permittivity $\varepsilon$, conductivity $\sigma$, and permeability $\mu$) between two conducting planes 
located at $z=0$ and $z=h$. Assume the waves have the following electric field 
polarization $E = (0, 0, E_z(x,y,z,t))$.

A) Use Maxwell’s equations to determine the dependence of $E_z$ on $z$. (Hint: $E_x$ and $E_y$ 
are zero.)

B) Derive the wave equation for $E_z$. What components of the magnetic field 
intensity $H$ are present?

C) Obtain expressions for the surface charge and surface current density in the top 
plate ($z=h$) in terms of the fields $E$ and $H$. Verify that charge is conserved and 
explain each term in the continuity equation.

D) Consider plane wave solutions of the form $E_z = \text{Re}\{A \exp[i(kx-\omega t)]\}$. What is the 
relation between $k$ and $\omega$? (hint: one or both may be complex)
3. (45 pts) For the model frequency dependent dielectric constant introduced in class:

\[ \varepsilon(\omega) / \varepsilon_0 = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}, \]

A) Calculate the dispersion relation for the electromagnetic and electrostatic modes supported by the medium.

B) Find the phase and group velocities of each branch as functions of frequency

C) Show that the expression

\[ W = \frac{1}{4} \int d^3x \frac{\partial \omega \varepsilon(\omega)}{\partial \omega} |\hat{E}|^2 \]

includes three contributions to stored energy. Identify each of the three modes where energy is stored