chapter 19

Reconsider energy associated with dipole distribution in an external field $\underset{\sim}{E}_0(\underset{\sim}{x})$

$$W = q\phi(0) - \underset{\sim}{p}\cdot\underset{\sim}{E} - \frac{1}{6}\sum_{ij} Q_{ij}\frac{\partial E_i}{\partial x_j}$$

suppose we have a <u>fixed</u> set of external charges ρ_{ext} which create an external field $\underset{\sim}{E}_0$

we now introduce a dielectric material with dielectric constant, in doing this, we assume ρ_{ext} does not change, however, ~~E will~~

the electric field will change from $\underset{\sim}{E}_0$ to some new value $\underset{\sim}{E}$. call

ΔW the difference in energy of the two states

$$\Delta W = \frac{1}{2}\int d^3x \left[\overset{\text{dielectric}}{\underset{\sim}{E}\cdot\underset{\sim}{D}} - \overset{\text{no dielectric}}{\underset{\sim}{E}_0\cdot\underset{\sim}{D}_0} \right]$$

where $\underset{\sim}{D}_0 = \overset{\epsilon_0}{\underset{\sim}{E}_0} \quad (\epsilon_0 = 1)$

$D = \epsilon E$ assume's linear,

skip

Re write

$$\Delta W = \cancel{\frac{1}{8\pi}} \int d^3x \left[\mathbf{E} \cdot \mathbf{D}_0 - \mathbf{D} \cdot \mathbf{E}_0 \right]$$

$$+ \frac{1}{8\pi} \int d^3x \, (\mathbf{E} + \mathbf{E}_0) \cdot (\mathbf{D} - \mathbf{D}_0)$$

note that $\quad \mathbf{E} + \mathbf{E}_0 = -\nabla(\phi + \phi_0)$

and thus

$$\frac{1}{8\pi} \int d^3x \, (\mathbf{E} + \mathbf{E}_0) \cdot (\mathbf{D} - \mathbf{D}_0)$$

$$= -\frac{1}{8\pi} \int d^3x \, (\phi + \phi_0) \underbrace{\left(\nabla \cdot (\mathbf{D} - \mathbf{D}_0) \right)}_{4\pi(\rho_{ext} - \rho_{ext}) = 0}$$

ρ_{ext} fixed

$$\Delta W = \frac{1}{8\pi}\int d^3x \left[\underset{\sim}{E}\cdot\underbrace{\underset{\sim 0}{D}}_{\epsilon_0 \underset{\sim}{E}_0} - \underbrace{\underset{\sim}{D}}_{\epsilon \underset{\sim}{E}}\cdot\underset{\sim 0}{E} \right]$$

$$= \frac{1}{8\pi}\int d^3x \,(\epsilon_0 - \epsilon)\,\underset{\sim}{E}\cdot\underset{\sim 0}{E}$$

$$4\pi \underset{\sim}{P} = \epsilon \underset{\sim}{E} - \epsilon_0 \underset{\sim}{E} = (\epsilon - \epsilon_0)\underset{\sim}{E}$$

induced polarazation density in final state

$$\Delta W = -\frac{1}{2}\int d^3x \; \underset{\sim}{P}\cdot\underset{\sim 0}{E}$$

external electric field

$\boxed{\text{O.K.}}$

compare with result for fixe dipole

*

$$W = q\,\phi(\underset{\sim 0}{x}) - \underset{\sim}{p}\cdot\underset{\sim}{E}(\underset{\sim 0}{x}) + \text{Quadrupole terms}$$

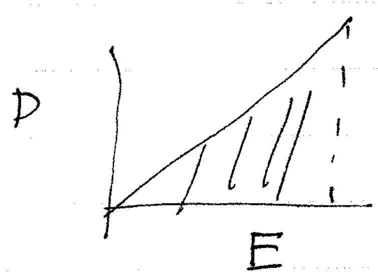
if $q = 0$ $\quad Q_{ij} = 0$

$$W = -\underset{\sim}{p}\cdot E \qquad\qquad \underset{\sim}{P} = N\underset{\sim}{p}$$

agree's except for a factor of 1/2

* assume $\underset{\sim}{p}$ is fixed independ of External field

actually if $\underset{\sim}{p} \propto \underset{\sim}{E}$ then



one gets $W = -\frac{1}{2}\underset{\sim}{p}\cdot\underset{\sim}{E}$

$$W = \int dW = -\int d\underset{\sim}{p}\cdot\underset{\sim}{E} = -\frac{1}{2}\underset{\sim}{p}\cdot\underset{\sim}{E}$$

Incremental Changes in Energy

Method of vitual displacements

supose I have a system of free charges ρ_f

and a dielectric ϵ

and I make a change to the system,

$$\rho_f \rightarrow \rho_f + \delta\rho_f$$

$$\epsilon \rightarrow \epsilon + \delta\epsilon$$

$$\underset{\sim}{E} \rightarrow \underset{\sim}{E} + \delta\underset{\sim}{E}$$

$$\underset{\sim}{D} \rightarrow \underset{\sim}{D} + \delta\underset{\sim}{D}$$

$$\underset{\sim}{D} + \delta\underset{\sim}{D} = (\epsilon + \delta\epsilon)(\underset{\sim}{E} + \delta\underset{\sim}{E})$$

$$\delta\underset{\sim}{D} \simeq \epsilon\,\delta\underset{\sim}{E} + \delta\epsilon\,\underset{\sim}{E} \qquad \text{to first order}$$

Change in energy

$$\delta W = \frac{1}{\cancel{8\pi}} \int d^3x \, \frac{\left[\delta\underset{\sim}{D} \cdot \underset{\sim}{E} + \underset{\sim}{D} \cdot \delta\underset{\sim}{E}\right]}{2}$$

can be written two ways

#1)

$$\underset{\sim}{D} \cdot \delta \underset{\sim}{E} = \underset{\sim}{E} \cdot \epsilon \delta \underset{\sim}{E}$$

$$= \underset{\sim}{E} \cdot (\delta \underset{\sim}{D} - \epsilon \delta \underset{\sim}{E})$$

$$\delta W = \frac{1}{\cancel{8\pi}\ 2} \int d^3x \left[2 \delta \underset{\sim}{D} \cdot \underset{\sim}{E} - \delta \epsilon |\underset{\sim}{E}|^2 \right]$$

now use $\underset{\sim}{E} = -\nabla \Phi$ $\quad \nabla \cdot \delta \underset{\sim}{D} = \cancel{4\pi}\, \delta \rho_f$

$$\delta W = \int d^3x\, \phi\, \delta \rho_f - \int d^3x\, \frac{\delta \epsilon}{8\pi} |\underset{\sim}{E}|^2$$

suppose no change in ρ_f $\quad \delta \rho_f = 0$

$$\delta W_{\cancel{1}Q} = - \int d^3x\, \frac{\delta \epsilon}{8\pi} |\underset{\sim}{E}|^2$$

~~suppose~~ pote

total change in energy

change in energy delivered by source

change in energy ma ϕ at fixed charge

$$\delta W = \int d^3x\, \phi\, \delta \rho_f + \delta W_Q$$

$\delta D = \cancel{\delta\epsilon}\ \epsilon\delta \underset{\sim}{E} + \delta\epsilon \underset{\sim}{E}$

~~suppose~~

#2 $\quad \delta \underset{\sim}{P}\cdot\underset{\sim}{E} = \delta\epsilon|\underset{\sim}{E}|^2 + \epsilon\delta\underset{\sim}{E}\cdot\underset{\sim}{E}$

$$= \delta\epsilon|\underset{\sim}{E}|^2 + \delta\underset{\sim}{E}\cdot\underset{\sim}{D}$$

$$\delta W = \frac{1}{\substack{\cancel{8\pi}\\ 2}}\int d^3x\left[2\,\delta\underset{\sim}{E}\cdot\underset{\sim}{D} + \delta\epsilon|\underset{\sim}{E}|^2\right]$$

$$\delta\underset{\sim}{E} = -\nabla\delta\phi \qquad \nabla\cdot\underset{\sim}{D} = \cancel{4\pi}\rho_f$$

$$\delta W = \int d^3x\,\rho_f\,\delta\phi + \int d^3x\,\frac{\delta\epsilon|\underset{\sim}{E}|^2}{8\pi}$$

suppose I make $\delta\phi = 0$ constant voltage

$$\delta W_V = \int d^3x\,\frac{\delta\epsilon|\underset{\sim}{E}|^2}{8\pi} = -\delta W_Q$$

$$\delta W = \delta W_V + \int d^3x\,\rho_f\,\delta\phi$$

Change in energy at fixed voltage

change in potential

Improved model of dielectric constant

$$\mathbf{P} = \underbrace{N}_{\text{number density of dipoles}} \langle \mathbf{p} \rangle = \frac{1}{\Delta V} \sum_{\substack{\text{molecules} \\ \text{inside } \Delta V}} \mathbf{p}_i$$

$\langle \mathbf{p} \rangle$: dipole moment of each one

suppose

$$\langle \mathbf{p} \rangle = \underbrace{\gamma_{mol}}_{\text{molecular polarizability}} \mathbf{E}$$

$$\mathbf{P} = N \gamma_{mol} \mathbf{E} = \underbrace{\chi_e}_{\text{susceptibility}} \mathbf{E}$$

$$\cancel{\epsilon = 1 + 4\pi N \gamma_{mol}}$$

$$\chi_e = N \gamma_{mol}$$

proportional to density

This assumes that each dipole acts independently of the others and responds only to the ~~average~~ local ~~electric fie~~ average electric field