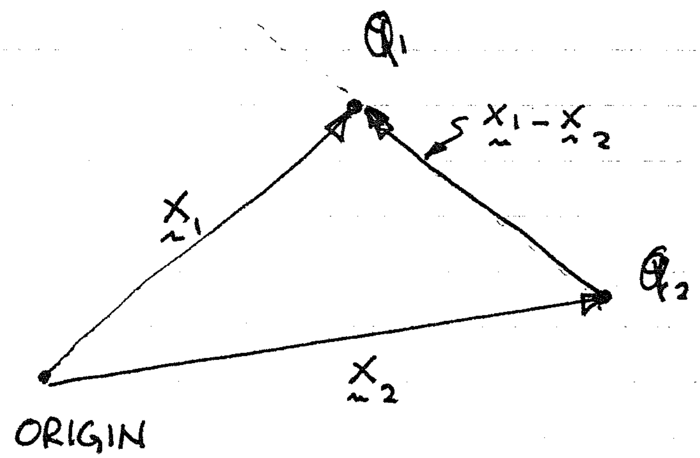
CHAPTER 1

Electrostatics

The fundamental law of electrostatics is Coulomb's law which describes the force acting betwen two point like charged bodies.



$\mathbf{F}$ = force on body # 1 due to body # 2

Vector notation

$$\underset{\sim}{\mathbf{F}} = k \frac{q_1 q_2 (\underset{\sim}{x}_1 - \underset{\sim}{x}_2)}{|\underset{\sim}{x}_1 - \underset{\sim}{x}_2|^3}$$

- inverse square law
- acts along ~~directi~~ line passing through bodies

- proportional to magnitude of each charge (charge is a quantifiable property of matter, similar to mass) however charge can be positive or negative

- assumes charges are stationary

- k is a constant of proportionality depends on system of units

Example: ~~MKS~~ ~~(SI)~~ MKSA — SI units

$\underset{\sim}{F}$ newtons

$\underset{\sim}{X}$ meters

Q, coulombs

1 coulomb $= 6.24 \times 10^{18}$ elementary charges

$$k = 8.988 \times 10^{9} \ \frac{\text{Newton meter}^2}{\text{coul}^2}$$

frequently one uses

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}$$

IF ONE USES cgs (esu) instead of MKS

$$1 \text{ Newton} = 1\text{KG} \cdot \frac{\text{meter}}{s^2} = 10^5 \text{ dynes}$$

$$\text{dyne} = \text{gram}\,\frac{\text{cm}}{s^2}$$

x in centimeters

~~coulom~~ Q (stick with coulombs for the moment)

$$\underset{\sim}{F}_{\text{dynes}} = k' \frac{Q_1 Q_2 (\underset{\sim}{x}_1 - \underset{\sim}{x}_2)}{|\underset{\sim}{x}_1 - \underset{\sim}{x}_2|^3}$$

$$k' = \frac{10^5 k}{(.01)^2} = 8.988 \times 10^{18}$$

Why should we stick with Coulombs?

lets pick the unit of charge to eliminate the constant of proportionality

(esu) electrostatic units

$$F_{dynes} = \frac{\underbrace{(k^{1/2} Q_1)}_{q_1} \underbrace{(k^{1/2} Q_2)}_{q_2} (\underset{\sim}{x}_1 - \underset{\sim}{x}_2)}{|\underset{\sim}{x}_1 - \underset{\sim}{x}_2|^3}$$

q_1 = charge in stat coulombs

~~1 stat coul~~

1 coulomb $= \sqrt{k'} = 2.998 \times 10^9$ stat coulomb

Note 1 stat coulomb $= \sqrt{\dfrac{cm^3\, gram}{sec}}$

~~Suppose a num~~

Aside
How is
Q defined?

Superposition Force due to ~~over~~ a number of charges is obtained by vector addition

$$\underset{\sim}{F}_1 = \sum_{j=2}^{N} \frac{q_1 q_j (\underset{\sim}{x}_1 - \underset{\sim}{x}_j)}{4\pi\epsilon_0 |\underset{\sim}{x}_1 - \underset{\sim}{x}_i|^3}$$

Electric Field

$$F_1 = q_1 \underbrace{\sum_j \frac{q_j (\underset{\sim}{x}_1 - \underset{\sim}{x}_j)}{4\pi\epsilon_0 |\underset{\sim}{x}_1 - \underset{\sim}{x}_j|^3}}_{\underset{\sim}{E}(\underset{\sim}{x}_1)}$$

call this the electric field at point 1. Here it is assumed that the charge is located at a point and the contribution of that charge to the electric field (which would be infinite is not included)

Here the first of many problems in electrodynamics appears. It will turn out that it is better ~~not to do when~~ in the frame work of classical electro-dynamics not to deal with point charges but rather to ~~consider cha~~ continuous charge distributions. The real resolution of ~~ass of~~ many of the problems associated with point charges ~~is resolved by quantum mechanics~~ lies outside the realm of classical physics.

continuous charge distribution

$\rho(\underset{\sim}{x})$ = charge density

~~stat~~ coul. / cm^3

the ~~to~~ infinitessimal charge residing in the small cube d^3x' centered at $\underset{\sim}{x}'$ is

$$dq' = \rho(\underset{\sim}{x}')\,d^3\underset{\sim}{x}'$$

The electric field produced by this charge distribution at point $\underset{\sim}{x}$ is given by

$$\underset{\sim}{E}(x) = \int d^3\underset{\sim}{x}'\,\frac{\rho(\underset{\sim}{x}')\,(\underset{\sim}{x}-\underset{\sim}{x}')}{4\pi\epsilon_0\,|\underset{\sim}{x}-\underset{\sim}{x}'|^3}$$

The force exerted on the infinitessimal charge in the small cube centered at $\underset{\sim}{x}$ is given by

Executive Summary Chapter/Sec 1.1–1.7

1 Force acting on charge #1 due to charge #2

$$\underset{\sim}{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2 (\underset{\sim}{x}_1 - \underset{\sim}{x}_2)}{|\underset{\sim}{x}_1 - \underset{\sim}{x}_2|^3}$$

2 Force due to many charges is super position

$$\underset{\sim}{F}_1 = \frac{1}{4\pi\epsilon_0} q_1 \sum_{j=2}^{N} \frac{q_j (\underset{\sim}{x}_1 - \underset{\sim}{x}_j)}{|\underset{\sim}{x}_1 - \underset{\sim}{x}_j|^3}$$

$$= q_1 \underset{\sim}{E}(\underset{\sim}{x}_1)$$

exclude point charge at $\underset{\sim}{x}_1$

3. If continuous charge densit

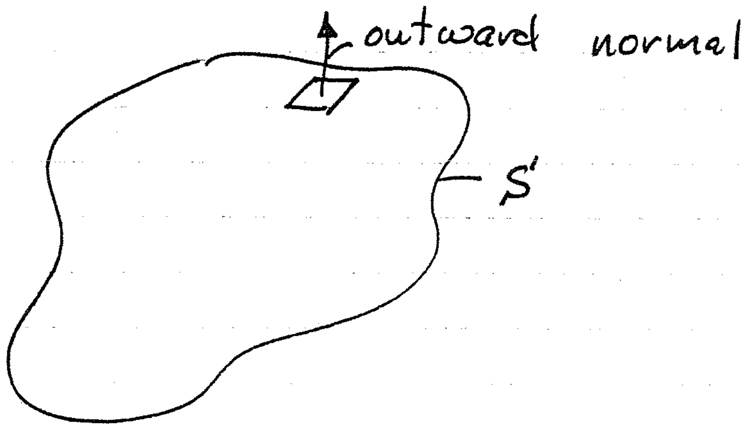
$$\underset{\sim}{E}(\underset{\sim}{x}) = \frac{1}{4\pi\epsilon_0} \int d^3x' \frac{\rho(\underset{\sim}{x}') (\underset{\sim}{x} - \underset{\sim}{x}')}{|\underset{\sim}{x} - \underset{\sim}{x}'|^3}$$

no longer need to worry about $\underset{\sim}{x} = \underset{\sim}{x}'$

$$d\underset{\sim}{F} = \rho(\underset{\sim}{x})\, \underset{\sim}{E}(\underset{\sim}{x})\, d^3x$$

Gauss's Law

Coulombs law + definition of electric field imply



S = closed surface $\quad V$ = enclosed volume

$$\epsilon_0 \int_S \underset{\sim}{E} \cdot \underset{\sim}{n}\, da = \cancel{4\pi} \int_V \rho(x)\, d^3x = \cancel{4\pi}\, q_{enclosed}$$

↑ outward normal

may be used to determine $\underset{\sim}{E}$ when

One can apply Gauss's law to an arbitrary small volume and obtain the differential form of Gauss's Law

$$\epsilon_0 \nabla \cdot \underset{\sim}{E} = \cancel{4\pi}\rho(\underset{\sim}{x})$$

DEFINITION of Divergence $\nabla \cdot \underset{\sim}{A} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \int_S da\, \underset{\sim}{n} \cdot \underline{A}$

$$\nabla \cdot \underset{\sim}{E} = \frac{\partial}{\partial x} E_x + \frac{\partial}{\partial y} E_y + \frac{\partial}{\partial z} E_z$$

in Cartesian Coordinates

(see book for other coordinate systems)

This also follows from the general vector relation "Divergence theorem"

$$\int_V d^3x\, \nabla \cdot \underset{\sim}{A} = \int_S \underset{\sim}{A} \cdot \underset{\sim}{n}\, da$$

for any closed surface S & vector field A.

Skip

How to Show $\boxed{1.3}$

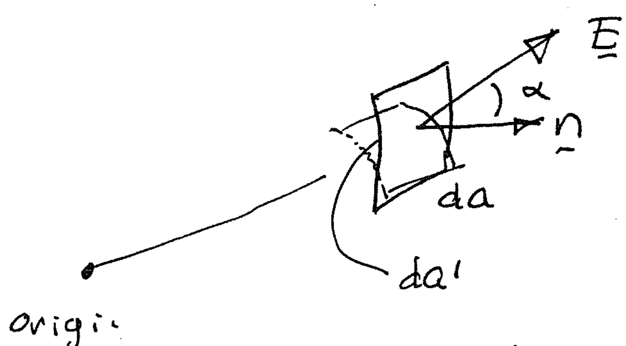
1. Show for one charge $\int_S \underset{\sim}{n} \cdot \underset{\sim}{E}\, da = ?$

2. Use superposition for arbitrary charge distributions

$$\underset{\sim}{E} = \frac{q}{4\pi\epsilon_0} \frac{\underset{\sim}{x} - \underset{\sim}{x}_i}{|\underset{\sim}{x} - \underset{\sim}{x}_i|^3}$$

take $\underset{\sim}{x}_i$ to be origin

$$\underset{\sim}{E} = \frac{q}{4\pi\epsilon_0} \frac{\underset{\sim}{x}}{|\underset{\sim}{x}|^3}$$



$$\underset{\sim}{n} \cdot \underset{\sim}{E}\, da = \frac{q}{4\pi\epsilon_0} \frac{da}{r^2} \cos\alpha$$

$$|da \cos\alpha| \equiv s\, da'$$

sign $\cos\alpha$

Portion of the surface of a sphere of radius r subtending same angle as da

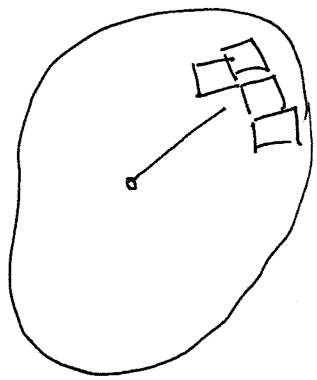
$$s \frac{da'}{r^2} = s\, d\Omega$$

differential solid angle (independent of r)

$$\int_S \underset{\sim}{n} \cdot \underset{\sim}{E}\, da = \int_S s\, d\Omega \frac{q}{4\pi\epsilon_0}$$

Now sum over all elements on surface

case # 1 surface surrounds charge $s = 1$

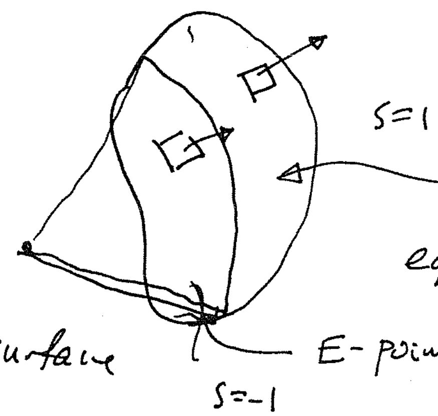


$$\int_S \frac{q}{4\pi\epsilon_0}\, d\Omega = \frac{q}{\epsilon_0}$$

$$\int_S d\Omega = 4\pi$$

why $4\pi r^2$ = area of surface $= \int da'$

case # 2



equal solid angles ≡ cancel

q out side surface

$$\int_S \frac{q s}{4\pi\epsilon_0}\, d\Omega = 0$$

Conclusion

$$\int_S \underset{\sim}{E} \cdot \underset{\sim}{n}\, da = \begin{cases} \dfrac{q}{\epsilon_0} & \text{if charge is inside} \\ 0 & \text{otherwise} \end{cases}$$

CHAPTER 2

Scalar Potential

The electric field ~~derivable~~ produced by a static distribution of charges can be ~~derived~~ expressed as the gradient of a scalar potential

$$\underset{\sim}{E} = -\nabla\phi(\underset{\sim}{x})$$

Follows from Coulomb's Law

not all vectors and in particular not all electric fields can be expressed this way. If the field is due to static charges then a scalar potential $\phi(\underset{\sim}{x})$ can be found

$$\nabla = \underset{\sim}{e}_x \frac{\partial}{\partial x} + \underset{\sim}{e}_y \frac{\partial}{\partial y} + \underset{\sim}{e}_z \frac{\partial}{\partial z} \quad \text{in cart}$$

Can be seen ~~fo~~ by noting

$$\nabla \frac{1}{|\mathbf{x}-\mathbf{x}'|} = -\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} \Longleftarrow \text{Show} \Longrightarrow$$

differentiate w.R.T

thus $\Rightarrow$ $$\mathbf{E}(\mathbf{x}) = -\int d^3x' \frac{\rho(\mathbf{x}')}{4\pi\epsilon_0} \nabla \frac{1}{|\mathbf{x}-\mathbf{x}'|}$$

$$= -\nabla \int \frac{d^3x' \rho(\mathbf{x}')}{4\pi\epsilon_0 |\mathbf{x}-\mathbf{x}'|} = -\nabla \phi(\mathbf{x})$$

~~denote~~

where

$$\phi(\mathbf{x}) = \int \frac{d^3x' \, \rho(\mathbf{x}')}{4\pi\epsilon_0 |\mathbf{x}-\mathbf{x}'|}$$

Potential due to a point charge q' at x'

$$\phi(\mathbf{x}) = \frac{q'}{4\pi\epsilon_0 |\mathbf{x}-\mathbf{x}'|}$$

Let's work this out

$$\nabla \frac{1}{|\underline{x}-\underline{x}'|} = \left(\underline{e}_x \frac{\partial}{\partial x} + \underline{e}_y \frac{\partial}{\partial y} + \underline{e}_z \frac{\partial}{\partial z} \right) \frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

$$\frac{\partial}{\partial x} \frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}} = -\frac{1}{\cancel{2}} \frac{\cancel{2}(x-x')}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}}$$

$|\underline{x}-\underline{x}'|^3$

$$\frac{\partial}{\partial y} = -\frac{(y-y')}{\cdots}$$

$$\nabla \frac{1}{|\underline{x}-\underline{x}'|} = - \frac{\underline{e}_x (x-x') + \underline{e}_y (y-y') + \underline{e}_z (z-z')}{|\underline{x}-\underline{x}'|^3} = - \frac{\underline{x}-\underline{x}'}{|\underline{x}-\underline{x}'|^3}$$

DIFFERENTIAL FORM

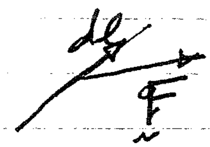
$$\epsilon_0 \nabla \cdot \underset{\sim}{E} = \cancel{4\pi}\rho(\underset{\sim}{x})$$

$$\underset{\sim}{E} = -\nabla\phi$$

$$\epsilon_0 \nabla^2\phi = -\cancel{4\pi}\rho(\underset{\sim}{x}) \qquad \textit{Poisson's Equation}$$

$$= 0 \qquad \textit{Laplace's Equation}$$

What is the physical significance of $\phi(\underset{\sim}{x})$ suppose we have an infinitessimal charge dq which we move through a static electric field. This requires Work



$$dW = \int_{\underset{\sim}{x}_1}^{\underset{\sim}{x}_2} d\underset{\sim}{F}\cdot d\underset{\sim}{l} = dq\int_{\underset{\sim}{x}_1}^{\underset{\sim}{x}_2} \underset{\sim}{E}\cdot d\underset{\sim}{l} = +dq\int_{\underset{\sim}{x}_1}^{\underset{\sim}{x}_2} d\underset{\sim}{l}\cdot\nabla\phi$$

moving from $\underset{\sim}{x}_1$ to $\underset{\sim}{x}_2$ $\quad = -dq[\phi(\underset{\sim}{x}_2)$

Stokes's Theorem

note, if $\underset{\sim}{x}_1$ and $\underset{\sim}{x}_2$ are the same

$\phi(\underset{\sim}{x}_2)=\phi(\underset{\sim}{x}_1)$ single value

$$\cancel{\oint \underset{\sim}{E}\cdot d\underset{\sim}{l} =}$$

$$\oint_{\text{closed path}} \nabla\phi\cdot d\underset{\sim}{l} = 0$$

This implies $\boxed{\nabla\times\nabla\phi=0}$

$$\nabla\times\underset{\sim}{A} = \underset{\sim}{e}_x\left[\frac{\partial}{\partial y}A_z-\frac{\partial}{\partial z}A_y\right]$$
$$\underset{\sim}{e}_y\left[\frac{\partial}{\partial z}A_x-\frac{\partial}{\partial x}A_z\right]$$
$$\underset{\sim}{e}_z\left[\frac{\partial}{\partial x}A_y-\frac{\partial}{\partial y}A_x\right]$$

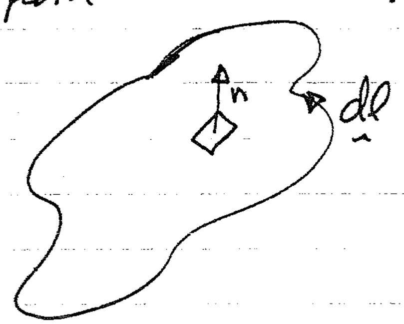
in cartesian

Stokes' theorem

unit normal

$$\oint_{\text{closed path}} \underset{\sim}{A}\cdot d\underset{\sim}{l} = \int_S (\nabla\times\underset{\sim}{A}\cdot\hat{n})\,da$$

any surface whose perimeter is the path



~~sense~~ sense of $\hat{n}$ defined by the direction of $d\underset{\sim}{l}$

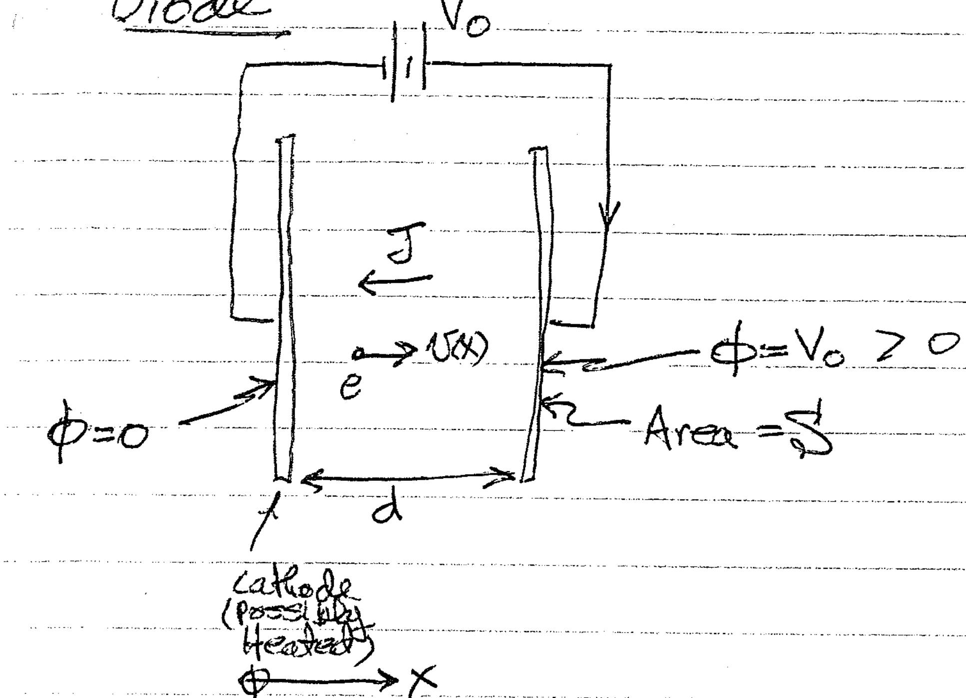
RIGHT HANDED rule

if $\underset{\sim}{A} = \nabla \phi$ this implies

$$\nabla \times \nabla \phi = 0 \Leftarrow \text{for any } \Phi$$

$\nabla \times \underset{\sim}{E} = 0$ for electrostatic fields

<u>Vacuum Diode</u>



J = current density $= I/S$ = constant ← We do not yet know J, We want to find it

$J = + n(x)\, e\, v(x)$

$\frac{1}{2} m v^2(x) - e\phi(x) = 0$

$$v(x) = \sqrt{\frac{2e\phi(x)}{m}}$$

$$-\rho(x) = e\, n(x) = J/v(x) = J\sqrt{\frac{m}{2e\phi(x)}}$$

Laplace's equation: $\nabla^2\phi = -\frac{\rho}{\epsilon_0}$

$$\frac{d^2\phi(x)}{dx^2} = \frac{J}{\epsilon_0}\sqrt{\frac{m}{2e\phi(x)}}$$ ρ depends non linearly on ϕ

Space charge limited emission.

<u>Boundary condition</u> $d\phi/dx = 0$ at $x = 0$

14'' ~~13¾~~

$$\frac{\phi}{x^2} \sim \frac{1}{\sqrt{\phi}}$$

$$\phi^{3/2} \sim x^2$$

$$\phi \sim x^{4/3}$$

Solution with

$$\phi(0) = 0$$

$$\frac{d\phi}{dx} = 0$$

$$\phi = \left[\frac{3}{4}\left(\frac{4J}{\epsilon_0}\right)^{1/2}\left(\frac{m}{2e}\right)^{1/4}\right]^{4/3} x^{4/3}$$

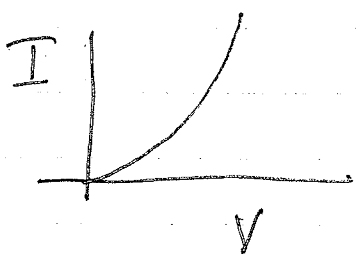
$$= \left[\left(\frac{3}{4}\right)^{4/3}\left(\frac{4J}{\epsilon_0}\right)^{2/3}\left(\frac{m}{2e}\right)^{1/3}\right] x^{4/3}$$

Now

Apply B.C. $\phi(d) = V_0 \Rightarrow$

$$J = \frac{4}{9}\epsilon_0\left(\frac{2e}{m}\right)^{1/2}\frac{V_0^{3/2}}{d^2}$$

Child – Langmuir law



$$I = \mu V_0^{3/2}$$

μ is called Perveance

$$\mu = \frac{4A}{9}\frac{\epsilon_0}{d^2}\left(\frac{2e}{m}\right)^{1/2}$$

4 Equivalent Formulation

$$\nabla \times \underline{E} = 0$$

$$\nabla \cdot \underline{E} = \rho(\underline{x})/\epsilon_0$$

$$\underline{E} \to 0 \quad \text{as} \quad |\underline{x}| \to 0$$

Solution of Poisson Equation

$$\nabla^2 \phi = -\frac{\rho(x)}{\epsilon_0} \qquad \begin{array}{l} \phi(0) = 0 \\ \phi(d) = V_0 \end{array}$$

$$\phi(x) = \frac{x}{d} V_0 + \int_0^d dx' \frac{\rho(x')}{\epsilon_0} G_{1D}(x, x')$$

$$G_{1D} = \begin{cases} x\left(1 - \frac{x'}{d}\right) & x' > x \\ x'\left(1 - \frac{x}{d}\right) & x' < x \end{cases}$$

PROBLEMS

Example

$$\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon_0}$$ known

Boundary conditions $\phi(x=0) = 0$

$$\phi(x=d) = V_0$$

Integrate once

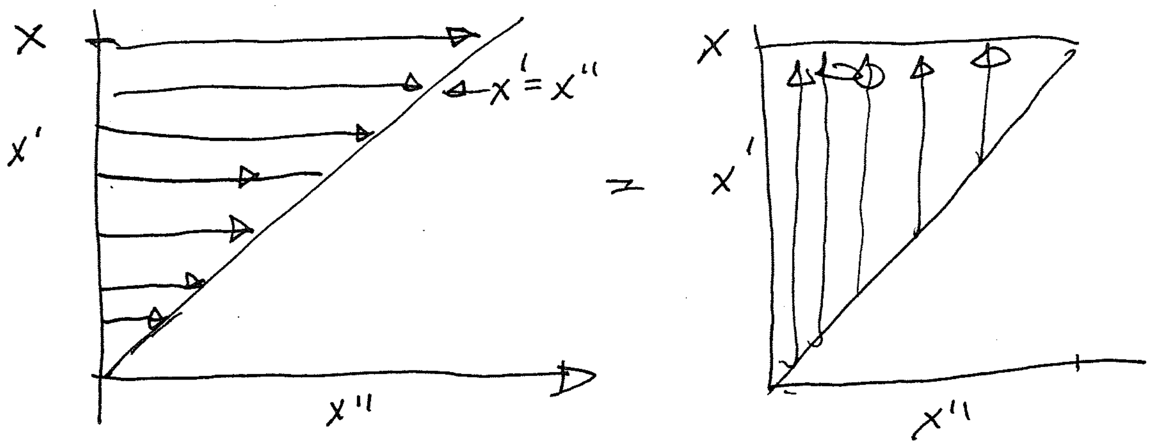
$$\frac{d\phi}{dx} = \left.\frac{d\phi}{dx}\right|_{x=0} - \int_0^x dx' \frac{\rho(x')}{\epsilon_0}$$

not known yet

$$\phi(x) = \overset{0}{\phi(0)} + \int_0^x dx' \frac{d\phi}{dx'}$$

$$\phi(x) = \int_0^x dx' \left[\left.\frac{d\phi}{dx}\right|_{x=0} - \int_0^{x'} dx'' \frac{\rho(x'')}{\epsilon_0} \right]$$

$$\phi(x) = x \frac{d\phi}{dx}\Big|_0 - \int_0^x dx' \int_0^{x'} dx'' \frac{\rho(x'')}{\epsilon_0}$$



$$\int_0^x \int_0^{x'} dx' dx'' = \int_0^x dx'' \int_{x''}^x dx' \frac{\rho(x'')}{\epsilon_0}$$

$$= \int_0^x dx'' \frac{\rho(x'')}{\epsilon_0} (x - x'')$$

$$\phi(x) = x \frac{d\phi}{dx}\Big|_0 - \int_0^x dx'' \frac{\rho(x'')}{\epsilon_0} (x - x'')$$

Apply BC at $x=d$

$$V_0 = \phi(d) = d\,\frac{d\phi}{dx}\Big|_0 - \int_0^d dx''\,\frac{\rho(x'')}{\epsilon_0}(d-x'')$$

Determines $\frac{d\phi}{dx}\Big|_0$

$$\frac{d\phi}{dx}\Big|_0 = \frac{V_0}{d} + \int_0^d \frac{dx''}{d}\,\frac{\rho(x'')}{\epsilon_0}(d-x'')$$

THUS

$$\phi(x) = \frac{x}{d}V_0 + \int_0^d \frac{dx''}{d}\,\frac{\rho(x'')}{\epsilon_0}(d-x'')\,x$$

$$- \int_0^x dx''\,\frac{\rho(x'')}{\epsilon_0}(x-x'')$$

LAST TERM

$$-\int_0^d dx''\,\frac{\rho(x'')}{\epsilon_0}\begin{cases} 0 & \text{if } x''>x \\ (x-x'') & \text{if } x''<x \end{cases}$$

D

$$\int_0^d dx'' \frac{\rho(x'')}{\epsilon_0} \begin{cases} (d-x'')\frac{x}{d} - 0 & x'' > x \\ (d-x'')\frac{x}{d} - (x-x'') & x'' < x \end{cases}$$

$$\cancel{x}\ x - \frac{x''x}{d} - x + x''$$

$$\cancel{(1-x}'' \quad x''$$

$$\int_0^d dx'' \frac{\rho(x'')}{\epsilon_0} \begin{cases} x\left(1-\frac{x''}{d}\right) & x'' > x \\ x''\left(1-\frac{x}{d}\right) & x'' < x \end{cases}$$

solution with no ρ

$$\phi(x) = \frac{x}{d} V_0$$

$$+ \int_0^d dx'' \frac{\rho(x'')}{\epsilon_0} G(x, x')$$

$$G(x, x') = \begin{cases} x\left(1-\frac{x''}{d}\right) & x'' > x \\ x''\left(1-\frac{x}{d}\right) & x'' < x \end{cases}$$

This is an example of a Green's function ~~I~~ exp

CHAPTER 3