ENEE 680 Final Exam Due Dec. 21, 2009

1) (40 pts.) Consider a disc shaped material or radius a and height d, which has permanent magnetization $M_0\hat{z}$. Here $\hat{z}$ is aligned with the axis of symmetry of the disc.

a) Formulate the problem of finding the magnetic field throughout space as a differential equation for a scalar potential χ where $\boldsymbol{H} = -\nabla\chi$.

b) Show that there is an effective magnetic surface charge density at $z = \pm d/2$, and $r < a$.

c) Based on your knowledge of electrostatics sketch the lines of H, B, and M.

d) Now calculate the fields in the plane z=0 approximately in the following cases. 1) a>>d and 2) a<<d.

2) (60 pts.) A system of oscillating free charge and current density creates an electric field, which after all transients have decayed is of the form

$$\boldsymbol{E}(\boldsymbol{x},t) = \boldsymbol{e}_x E_0 \exp(-\sigma r^2) \cos(kz-\omega t)$$

where σ, k, and ω are specified parameters. Assume the medium is described by linear constituative relations involving $\mu = \mu_0$ and $\varepsilon = \varepsilon(\omega)$ where $\varepsilon(\omega)$ is a known function of frequency.

a) Find the corresponding magnetic field intensity, $\boldsymbol{H}(\boldsymbol{x},t)$.

b) Plot the phasor amplitude at z=0 of the x, y and z components of $\boldsymbol{E}$ and $\boldsymbol{H}$ as functions of x (for y=0) and y (for x=0)

c) Take the limit $\sigma/k^2 << 1$ (This assumption applies to the remainder of this problem), and find the free current density that is creating these fields. For what value of ω is the current density very small, explain your result.

d) Assume ε is real and calculate the average energy per unit length in z associated with the i) electric and ii) magnetic fields. iii) What is the power flowing through the plane z=0 ?

e) Now assume that $\varepsilon = \varepsilon_r + i\varepsilon_i$, and calculate the power per unit length in z transferred from the free current to the fields. Where does this power go?