1. A 10 V battery is connected through a 100 Ω resistor to a 100 meter length of 50 Ω transmission line. The line is terminated in an open circuit. At t=0 the battery is disconnected and the 100 Ω resistor is connected across the line. The speed of propagation on the line is $2.5 \times 10^8$ meters/sec.

$$R_0 = 100 \ \Omega$$

$$T = \frac{100}{2.5 \times 10^8} = 0.4 \ \mu\text{sec}$$

$$Z_0 = 50 \ \Omega$$

$$V_0 = 10 \ \text{V}$$

$$100 \ \text{m}$$

a) What are the voltage and current on the line just before the battery is disconnected? (5 points)
b) Plot the voltage and current on the line versus z for i) $t = 0.2 \ \mu\text{sec}$ and ii) $t = 0.6 \ \mu\text{sec}$ (15 points)
c) How much energy is dissipated in the source resistor between $t=0$ and $t = 0.6 \ \mu\text{sec}$? (10 points)

\[a) \ V = 10 \ \text{V} \quad I = 0 \quad V = 10 + V_p \quad I = 0 + I_p \quad I_p = \frac{-10}{100} = -0.1 \ \text{A}\]

\[b) \ V = 500 \ \text{mV} \quad 1 \ \text{V} \quad 5.66 \ \text{V} \quad 6.66 \ \text{V} \quad 3.33 \ \text{V} \quad c) \ U = 0.6 \times 10^4 \ \text{J} \]

\[U = 0.6 \times 10^4 \left(\frac{1}{15}\right)^2 \times 100 \quad U = 2.44 \times 10^{-6} \ \text{J}\]
2. A 100 ohm lossless transmission line is driven by a 100 MHz source. The speed of propagation on the line is $2 \times 10^8$ meters/sec. Attached to the end of the line is a load of impedance $43 - j32$ ohms. Using pieces of the given transmission line, design a single stub matching section so that all the incident power is absorbed in the load. Give specific numbers for the length and attachment position of the shorted stub. Label specific points on the attached Smith Chart. (30 points)

$$\lambda = \frac{2 \times 10^8 \text{m/s}}{10^8 \omega \cdot \text{Hz}} = 2\text{m}$$

$$0.15\lambda = 1.3\text{m}$$

$$\frac{3}{8}\lambda = 0.75\text{m}$$
\[
\frac{Y_{eq}}{V_0} = 1 - j1
\]

\[
\begin{align*}
\frac{0.34}{0.19} &= \frac{0.15}{0.13}
\end{align*}
\]
2) Radiation with free space wave length \( \lambda = 7.48 \times 10^{-2} \) meters is incident at \( 45^\circ \) on a \( L = 10^{-2} \) meter thick slab of dielectric material with dielectric constant \( \varepsilon = 4 \varepsilon_0 \). The electric field of the incident radiation is in the plane of incidence.

\[
Z_1 = \frac{Z_0 + jZ_0 \tan \beta L}{Z_0 + jZ_0 \tan \beta L}
\]

\[
Z_m = \frac{Z_0 + Z_0 \cos \beta L + jZ_0 \sin \beta L}{Z_0 + Z_0 \sin \beta L} = \frac{Z_0 + jZ_0 \tan \beta L}{Z_0 + jZ_0 \tan \beta L}
\]

a) What are the cosines of the angles of propagation with respect to the normal for the waves in each of the three regions? (10 points)

b) What are the wave impedances in each of the three regions. (10 points)

c) What are the fractions of power reflected from and transmitted through the slab? (10 points)

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\[
\sin \Theta_1 = \sin \Theta_2 = \sin \Theta_3 = \frac{1}{\sqrt{2}}
\]

\[
\cos \Theta_1 = \sqrt{1 - \sin^2 \Theta_1} = \frac{1}{\sqrt{2}}
\]

\[
Z_m = \sqrt{\frac{\varepsilon_0}{\varepsilon}} \cos \Theta_1
\]

\[
Z_1 = Z_3 = \frac{2\pi}{\lambda} \sqrt{\frac{\varepsilon_0}{\varepsilon}} L = 267 \, \Omega
\]

\[
Z_2 = \frac{2\pi}{\lambda} \sqrt{\frac{\varepsilon_0}{\varepsilon}} = 176 \, \Omega
\]

\[
k_2 = \sqrt{\frac{2\pi}{\lambda} \cos \Theta_2} \Rightarrow a \cdot \frac{2\pi}{\lambda} \sqrt{\frac{\varepsilon_0}{\varepsilon}} L = k_2 L = 1.57 = \frac{\pi}{2}
\]

\( \hat{\theta} = 0 \quad T = 1 \)
4. A rectangular air-filled waveguide of dimensions a=4 cm and b=2 cm carries RF power in the lowest mode of propagation.

a) What is the cut-off frequency for this mode (Assume \( \varepsilon = \varepsilon_0 \) for air.) (5 points)
b) What two modes have the next lowest cut-off frequencies? (5 points)
c) Assuming the long transverse direction is the x direction, write expressions for the spatial dependence of \( E_z \) and \( H_z \) for this mode. (15 points)

\[
\begin{align*}
\hat{E}_z &= \frac{-1}{k^2 - k_z^2} \left[ jk_z \nabla_x \hat{E}_z - j\mu_0 \varepsilon \times \nabla_x \hat{H}_z \right] \\
\hat{H}_z &= \frac{-1}{k^2 - k_z^2} \left[ jk_z \nabla_x \hat{H}_z + j\varepsilon \times \nabla_x \hat{E}_z \right]
\end{align*}
\]

d) If the frequency is twice the cut-off frequency what are the phase and group velocity? (5 points)

\[
\omega_c = \frac{1}{\varepsilon_0 \mu_0} \cdot \frac{\pi}{\alpha} \\
f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \cdot 4 \times 10^{-2}} = \frac{3}{8} \times 10^8 = 3.75 \times 10^8 \text{ Hz}
\]

(\( \varepsilon_0 = \frac{1}{\mu_0} \) is the free space constant)

\[
\begin{align*}
\frac{\gamma}{c} &= \frac{\pi}{\alpha^2} \\
\frac{\omega}{c} &= \sqrt{\frac{\pi}{\alpha^2}} \\
\frac{\omega}{c} &= \sqrt{\frac{\pi}{\alpha^2}} \\
\frac{\omega}{c} &= \frac{\pi}{\alpha} \\
\frac{\omega}{c} &= \frac{\pi}{\alpha}
\end{align*}
\]

\[
\begin{align*}
TE_{20} &= TE_{01} \Rightarrow \frac{f}{c} = 7.5 \times 10^8 \text{ Hz}
\end{align*}
\]

\[
\begin{align*}
\frac{\gamma}{c} &= \frac{\pi}{\alpha} \\
\frac{\omega}{c} &= \sqrt{\frac{\pi}{\alpha^2}} \\
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\end{align*}
\]

c) \( TE_{10} \Rightarrow \quad E = \hat{y} E_0 \sin\left(\frac{\pi x}{a}\right) \)

d) \( \frac{V_p}{V_0} = C \frac{\omega_c}{\sqrt{1 - \frac{\omega^2}{\omega_c^2}}} = \frac{3 \times 10^8}{\sqrt{\frac{3}{9}}} = \frac{3 \times 10^8}{\sqrt{3}} = 3.464 \times 10^8 \text{ m/s} \)

d) \( \frac{V_g}{V_0} = C \frac{\omega_c}{\sqrt{1 - \frac{\omega^2}{\omega_c^2}}} = \frac{\sqrt{\frac{3}{9}} \times 10^8}{\sqrt{\frac{3}{9}}} = 2.598 \times 10^8 \text{ m/s} \)