Smith Chart

Graphically solves the following bi-linear formulas

\[ \frac{Z_{eq}(l)}{Z_0} = \frac{1+\left(\rho e^{-2jkl}\right)}{1-\left(\rho e^{-2jkl}\right)} \]

\[ \rho = \frac{(Z_L / Z_0) - 1}{(Z_L / Z_0) + 1} \]

Note: works for admittance too.

\[ \frac{Y_{eq}(l)}{Y_0} = \left( \frac{Z_{eq}(l)}{Z_0} \right)^{-1} = \frac{1-\left(\rho e^{-2jkl}\right)}{1+\left(\rho e^{-2jkl}\right)} \]

Just switch sign of \( \rho \)

\[ \rho \rightarrow -\rho \]
Smith chart is the interior of the unit circle in the complex plane.

Example:

\[ \rho = \frac{1}{2} e^{j \frac{\pi}{4}} \]
Find $Z_L$ given $\rho$

\[
\frac{Z_{eq}(l)}{Z_0} = \frac{1 + (\rho e^{-2jkl})}{1 - (\rho e^{-2jkl})}
\]

Note: $Z_{eq}(l = 0) = Z_L$

Find real and Imaginary parts:

\[
\frac{Z_L}{Z_0} = \frac{1 + \rho}{1 - \rho} = R + jX
\]
Curves of constant real part: \( R \)

\[
\frac{Z_L}{Z_0} = \frac{1 + \rho}{1 - \rho} = R + jX
\]
Curves of constant imaginary part: X
\[ Z_L = \frac{1 + \rho}{1 - \rho} = 1.4 + j1.3 \]
What is $Z_{eq}$ at $l = \lambda/4$ from the load?

$$\frac{Z_{eq}(l)}{Z_0} = \frac{1 + (\rho e^{-2jkl})}{1 - (\rho e^{-2jkl})}$$

$$\frac{Z_L}{Z_0} = \frac{1 + \rho}{1 - \rho} = 1.4 + j1.3$$

$$\frac{Z_{eq}}{Z_0} = \frac{1 + \rho e^{-j\pi}}{1 - \rho e^{-j\pi}} = 0.38 - j0.34$$
Sample Problem: find $Z_{eq}$

$$f = 5 \text{ MHz} \quad Z_0 = 100 \, \Omega, v = 10^8 \, \text{m/s}$$

$$Z_L = (70 + j50) \, \Omega$$

$$Z_{eq} = ?$$

Method 1:

$$Z_{eq}(l) = Z_0 \frac{Z_L \cos kl + jZ_0 \sin kl}{Z_0 \cos kl + jZ_L \sin kl}$$

$$k = 2\pi / \lambda$$

$$\lambda = v / f = 10^8 / 5 \times 10^6 = 20 \text{ m}$$

$$kl = 0.628$$

or

$$l / \lambda = .1$$

$$\cos kl = 0.81$$

$$\sin kl = 0.59$$

$$Z_{eq}(2) = Z_0 \frac{56.63 + j99.5}{51.50 + j41.14}$$

$$Z_{eq}(2) = (161 + j64) \Omega$$
Method 2: Smith Chart

\[
\frac{Z_L}{Z_0} = \frac{(70+j50)}{100} \\
\frac{Z_L}{Z_0} = (.7+j.5)
\]

Move .1 \( \lambda \) toward generator

\[
\frac{Z_{eq}}{Z_0} = (1.6+j.65)
\]
Standing Wave Problem

\[ Z_L = ? \]

\[ \lambda = 20 \text{m}, \quad Z_0 = 100 \ \Omega \]

\[ V_{SWR} = 3 \]

\[ \lambda = 20 \text{m}, \quad Z_0 = 100 \ \Omega \]

\[ \frac{Z_{eq}(l_{max})}{Z_0} = \frac{1 + (\rho e^{-2jkl_{max}})}{1 - (\rho e^{-2jkl_{max}})} \]

\[ \frac{Z_{eq}(l_{max})}{Z_0} = \frac{1 + |\rho|}{1 - |\rho|} = VSWR \]

1.7+j1.3
\[ Z_L = 170+j130 \ \Omega \]

\[ l_{max} \]
f = 5 MHz \hspace{1cm} Z_0 = 100 \ \Omega, \nu = 10^8 \ \text{m/s}

Z_L = (140+j130) \ \Omega

Matching

\frac{Z_L}{Z_0} = (1.4+j1.3)

\frac{Y_L}{Y_0} = (0.38-j0.34)

\frac{Y_{eq}}{Y_0} = (1.0+j1.2)

\frac{Y_s}{Y_0} = (0-j1.2)

\frac{l}{\lambda} = 0.06 + 0.168 = 0.228
Shunt admittance

Short Circuit Impedance

\[ Y_s/Y_0 = (0-j1.2) \]

Short Circuit Admittance

\[ \lambda = 0.11 \]