* Show Gauss' Law Applies for a Single Point Charge

We can without loss of generality place the point charge at the origin of a spherical coordinate system. The closed surface is defined by $R = R_s(\theta, \phi)$.

The evaluation of $\int E \cdot ds$ should then become an integral over $\theta, \phi$.

At each point on the arbitrary closed surface we can say

$$E \cdot ds = E' \cdot ds'$$

where $ds'$ is a small rectangular portion of the surface of a sphere or radius $R_s(\theta, \phi)$ subtended by the angles $d\theta$ and $d\phi$. This takes geometry to show but is true.

$$E' \cdot ds' = E_r \frac{r^2 \sin \theta d\theta d\phi}{dA'}$$

So

$$E \cdot ds = \frac{q}{4\pi \varepsilon_0} \int_0^\theta \int_0^{2\pi} \frac{r^2 \sin \theta d\theta d\phi}{4\pi \varepsilon_0} = \frac{q}{16\varepsilon_0}$$

$$\int_E E \cdot ds' = \frac{q}{16\varepsilon_0}$$
What if \( q \) is not inside the surface?

There will be two contributions to the integral from each value of \( \Theta \) of \( \Phi \).

These will cancel

\[
d s_2 \cdot E_2 = \frac{q}{4 \pi \epsilon_0} \sin \Theta \sin \phi \cos \phi \\
d s_1 \cdot E_1 = -\frac{q}{4 \pi \epsilon_0} \sin \Theta \sin \phi \cos \phi
\]